# Graphicacy issues in school textbooks and designing learning contexts to address them

A Synopsis of Ph.D. Thesis

Amit Dhakulkar
Homi Bhabha Centre for Science Education,
Tata Institute of Fundamental Research

1st August 2017

# Contents

# Abstract

The present work is about graphs and their use in the context of science education. Graphs are a very powerful form of inscription, which enable communication, visualisation and analysis of data. Education researchers report that the ability to work with graphical representations is underdeveloped among learners. Surprisingly, in spite of the centrality of graphical representations across different subjects, there has been no systematic study on their use and efficacy in the Indian pedagogic contexts. We carried out an inquiry to assess the use of graphs in NCERT textbooks for science, maths and other subjects. Based on the analysis of this preliminary study, we first provide a critique of the shortcomings of graph use, using various established criteria commonly used by educational researchers and then devise and test pedagogic strategies to determine good practices with respect to educating young learners about graphs. We address this problem along two themes in our work. What is currently being done to enhance the ability to handle graphs successfully? What kind of experiences will provide learners with the ability to handle graphs successfully?

The structure of the document reflects the structure of the main thesis. Each section in this document reflects a chapter in the thesis. We have covered the important themes and topics, and have left out the details. The thesis has two parts: the first part situates the idea of graphicacy in the context of education and its associated problems as reported in literature. The motivation for this study, the importance of graphs in science and science education, and the research problems for the current work are defined (**Chapter 1**). A critical review of literature for problems of comprehending and constructing graphs and various models of graph comprehension are reported (**Chapter 2**). The review of textbooks gives us the overview of graphical practices in the Indian context (**Chapter 3**). We conclude the first part with problems of graphicacy and their possible solutions.

The second part presents a theoretical framework to develop activities which address the issues raised in the first part. The framework is based on our own analysis and implications from the literature (**Chapter 4**). The activities developed on the basis of this framework, their field testing and analysis are detailed (**Chapter 5**). The last chapter discusses the major outcomes of the work, with limitations and scope for future work (**Chapter 6**).

The appendices include: (**A**) Data from the textbook analysis, (**B**) A Brief historical survey of the origin of graphs, and (**C**) Brief description of the field work which was not analysed in detail.

# Part 1: Situating Graphicacy and its Problems

In the first part *Situating Graphicacy and its Problems* we introduce the concept of graphicacy as a generic skill required for making sense of data which may be generated from experiments or observations.

## 1 A case for Graphicacy

### Motivation for this study

In today's information centric world, we live with graphics or images across media types. In the ever expanding multi-media culture we find popular media, such as, television, newspapers, magazines etc. making use of graphics widely. In the context of science it is almost impossible to find a journal or a textbook on science which does not have a variety of graphics. We encounter different types of graphics like photographs, illustrations, graphs, paintings, sketches, maps, diagrams among others. Each of these types has its own potential and performs a set of functions. A particular graphic may perform more than one function, and in some cases it is almost impossible to achieve the same result without the graphic. Processing information in the present society which is full of technological artefacts is highly dependent on the reader's ability to comprehend graphs (Curcio, 1987).

But making sense of graphics is not natural. For each of the types we have mentioned there is a way of understanding them, performing operations and also making them. This skill of understanding graphics has to be taught explicitly. The term *graphicacy*, coined by Balchin & Coleman (1966) carries this connotation - the skill to understand graphics in general. Aldrich & Sheppard (2000) define graphicacy as:

> the ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats.

In our work we focus on graphs which represent quantitative data that help build mathematical models.

In scientific literature graphs are widely used to re-present data. (Cleveland, 1984; Krohn, 1991). These re-presentations in turn inform and influence fact construction, theory testing, and the intermediate process of theory formation (Smith, Best, Stubbs, Johnston & Archibald, 2000). It is virtually impossible to find a scientific manuscript or textbook without graphs (Roth & Bowen, 2003). Furthermore, among the various graphical representations found in the scientific discourse, graphs constitute the dominant form (Roth, Bowen & McGinn, 1999). Given the preference for graphs in scientific discourse, we collate, from various published research articles, a comprehensive set of functions a graph may satisfy. We broadly classify the information in the following four categories:

§ **Communication of Data**: To communicate and present the data in a parsimonious way.

§ **Analysis of Data**: To study the data for patterns and trends.

§ **Modeling**: To create mathematical models for explaining data and phenomena.

§ **Rhetorical Device**: To argue for a particular conclusion.

Graphs provide several affordances to those who can read them well. A well trained scientist is adept at grasping the underlying phenomenon while engaging with abstracted representation. Arguably, this ability influences multiple activities that a working scientist engages with namely, making hypothesis, correlating disparate observations, designing experiments, and drawing inferences. There are several instances in history of science where graphs have played an important role, leading to new discoveries and better theories. For example, discovery of dark matter in the galaxies from galactic rotation curves.

Graphs also play a crucial role in everyday discourse, particularly in mainstream media. The centrality and pervasiveness of graphs in science led Latour (Latour & Woolgar, 1986) to conclude that scientists exhibit a "graphical obsession", and to suggest that, in fact, *the use of graphs is what distinguishes science from non-science.* Perhaps this misperception that any discourse containing graphs is scientific, leads to many commercial advertisements to exploit a 'scientific' basis for selling. Studies show that advertisements become more convincing when graphs are used (Tal & Wansink, 2016). There are several instances of graphs used to obfuscate data, make misleading claims on many issues and matters relating to public (Wainer, 1984). This social reality further underscores the need to ensure education of the general public in understanding graphs. Given that graphs are part of standard curriculum in our existing school system, it would be highly desirable to inculcate critical understanding of graphs from early learning stages.

In *Critical Graphicacy*, Roth, Pozzer-Ardenghi & Han (2005) discuss graphs and their usage in textbooks. They found both qualitative and quantitative differences between the uses of graphs in textbooks and in scientific journals. The number of Cartesian graphs that are used in the textbooks was found to be low as compared to that in the journals. In textbooks only a few of the graphs had data from actual experiments. Frequently the graphs lacked basic features like units, scales and captions and many graphs were not even mentioned in the main texts. This made the interpretation of the graphs difficult not only for students, but also for experts. In contrast, scientific journals provide detailed help to understand the meaning of the graph and the graphs are usually very well integrated with the main text. Knowledge about graphical representation of the phenomenon to be presented is assumed by the authors of the textbooks. So the problem for the students becomes *two-fold*: on the one hand they are identified in the literature as lacking in skills required to comprehend and construct graphs, and on the other hand the textbooks do not provide them with enough resources to read and interpret graphs. Little is currently being done to directly address this issue (Aldrich & Sheppard, 2000; Peden & Hausmann, 2000; Paoletti, 2007). When present, the tasks on graphs are elementary in nature, with little emphasis on qualitative, investigative or critical questions (Leinhardt, Zaslavsky & Stein, 1990; Padilla, McKenzie & Shaw, 1986; Brasell & Rowe, 1993)

We have seen the definition of *graphicacy*, we now define *graphicate*. To graphicate is: to understand the phenomena behind the graph, to interpret and construct the meaning of various features and nuances of the graph, to predict, to extrapolate the data, to make conjectures, to make a model describing the phenomena, to ask critical questions, to construct a graph, or to describe the phenomena in the graph. We present our findings on how Indian science textbooks aid learners to become graphicate in Chapter 3. In this thesis, we address the following two sets of questions in the context of Indian school science textbooks:

**Part 1: Analysis of the school textbooks with graphs**

1. How are graphs placed in the Indian school textbooks in different subjects and different classes?
2. What kind of opportunities do Indian school science textbooks offer to learners to engage with graphs meaningfully? What are the missed opportunities which could be used effectively?

**Part 2: Developing and testing the activities**

3. What kind of learning design can enhance ability to construct and comprehend graphs?

4. What kind of experiences and technological tools will help learners to become graphicate?

The questions above are addressed in the two sections of the thesis respectively. In *Part 1: Situating Graphicacy and its Problems*, Chapter 3 we address questions 1 and 2 in the parts on quantitative and qualitative analysis respectively.

In *Part 2: Learning Contexts: Design, Development, Testing and Outcomes* we address questions 3 and 4. The remaining part of the synopsis presents highlights from each chapter.

# 2 Problems with graphs

The importance of constructing and comprehending graphs as a core skill for science and mathematics is well established. However learners face a lot of challenges in both comprehension as well as construction of graphs. This problem is compounded by lack of meaningful opportunities in the textbooks. We review the main outcomes from the earlier work done in this area.

The problems with graphs can be broadly divided into two categories: comprehension and construction. Graph comprehension is a complex activity. There are many factors involved in correctly reading a graph. Comprehension or interpretation of graphs would indicate the reader extracting meaning, mathematical relationship or making sense of situation or phenomena depicted. Some of the major factors on which research studies have been done include prior domain knowledge of the learners, prior experience of reading graphs, design of the graph, context in which the graphs are set, and cognitive models and sociological models for understanding of graphs. A successful comprehension of a graph involves an interaction of many factors. A graph comprehension can be said to be successful if the reader[1] can answer a certain set of questions based on the graph. Depending on the nature of these questions and the processing required to answer them, they can be categorised by the levels of comprehension as elementary, intermediate or advanced (Curcio, 1987; Wainer, 1992). Literal reading of graphs, that is, elementary level questions, do not present much problems, but with interpretation questions students face difficulties (Pereira-Mendoza & Mellor, 1990). In construction of graphs the learner has to make sense of data, and choose an appropriate way of displaying the data. In this process the learner has to create new structures which have relationship, and represent the data given in different formats to the graphical (Leinhardt et al., 1990).

In science, constructing graphs is a means of extracting knowledge from data. The construction of graphs involves the prior knowledge of the reader about the topic of graphs, the nature of graphs, design of graphs among other things. In some cases flexible guidelines for learners can be helpful in drawing the meaning from the graph. For example, Tufte (2001) provides a list of features a good graph should posses; Deacon (1999) gives a five step guide for drawing graphs from experimental data; Cleveland (1984) provides a set of guidelines for authors and editors of academic journals; Shah, Mayer & Hegarty (1999) provide guidelines for presenting graphs in textbooks. In another line of

[1] Reader here means the person who is viewing and interpreting the graph. Learner and/or student will be used interchangeably with reader.

thought it is claimed that constructing graphs with a purpose in a familiar context might actually help learners to understand graphs better (Ainley, 1995).

In the process of reading graphs, readers make systematic errors. These errors might have their origin in the prior experience or the lack of it. While reading graphs, domain knowledge about the phenomena depicted by the graph, design of the graphs or possessing different models about features of the graphs play an important role in comprehension and construction. There are cognitive models which attempt to explain these errors. For example, Physics Education Research (PER) provides us with some of the common misconceptions that students have while reading graphs in the context of physics.

The concept of motion presents several opportunities to introduce the idea of graphs in a natural way. It is not surprising that several studies related to graphs have been done on the topic of motion. We also note in our textbook analysis (Section 3) the topic of motion uses maximum number of graphs in science textbooks. The learners prior knowledge and experience in dealing with graphs matter significantly in their performance. Many misconceptions about ideas of motion in the context of graphs have been reported (McDermott, Rosenquist & van Zee, 1987; Beichner, 1994; Kozhevnikov & Thornton, 2006; Wemyss & van Kampen, 2013; Eshach, 2014). Nachmias & Linn (1987) looked at aspects of graph scaling, hardware and software issues and experimental variation in the context of cooling and heating phenomena. Dori & Sasson (2008) explore graphing in context of computer base chemistry learning environment. In life sciences the studies with graphs as focus include Adams & Shrum (1990), Pechenik Jan A (1992), Roth et al. (1999), Tairab, Khalaf & Ali (2004). Phillips (1997) discusses a study with primary level learners use of computers for handling graphs. In mathematics education the focus is on understanding relation of graphs of algebraic functions to other representations like algebraic and tabular and graphs depicting various situations (Leinhardt et al., 1990; Even, 1993; Moschkovich, Schoenfeld & Arcavi, 1993; Moschkovich, 1996; Mevarech & Kramasky, 1997; Hitt, 1998; Even, 1998). Some studies also focus on the ability of students to represent situations using graphs (Bell & Janvier, 1981). Asiala, Cottrill, Dubinsky & Schwingendorf (1997) provide a detailed deconstruction of epistemological concepts linked to derivative. Another aspect of mathematics education studies focuses on technologies like graphing calculators and computer applications, in constructing and comprehending graphs. Bowen & Roth (2005) discuss the understanding of graphs in practice by pre-service teachers, they found that these teachers seem ill prepared to teach data collection and analysis in the way suggested by reform documents. González, Espinel & Ainley (2011) discuss graphical competence in statistical context. Anscombe (1973) and D. R. Cook & Weisberg (1999) consider various types of graphs from a statistical context.

The common problems or misconceptions that students have while reading graphs are shown in Table 1.

- § Confusing slope with height to arrive at wrong conclusions
- § Unable to interpreting changes in height and changes in slope
- § Difficulty in interpreting negative values of variables
- § Unable to relating one type of graph to another
- § Matching narrative information with relevant features
- § Not able to interpret the area under a graph
- § Conceiving a graph as a picture or a map
- § Confusing an interval and a point
- § Conceiving a graph as constructed of discrete points
- § Misreading the scale and variables on them
- § Confusing between variables when scales on graphs are changed
- § Being able to find slope of lines passing through origin but not otherwise
- § Looking for information that cannot obtained from the graph
- § Lack of knowledge about interpretive sources and familiarity

Table 1: Problems reported in literature on comprehending graphs. Detailed discussion about these topics is provided in Chapter 2 of the thesis.

Many of these problems were noted in our own studies during the development and field testing of the activities. Detailed discussions on these are presented in the respective chapters of the activities.

Similarly for graph construction some of the reported problems or misconceptions are listed in Table 2. Though some of the problems listed are taken care of when computers are used for graphing the data, the presence of automated graphing presents problems of its own. Tufte is particularly critical of using default graphic formats provided by computer applications indiscriminately for graph construction.

## Models of Graph Comprehension

The studies in psychological and cognitive research try to find which types of graphs are more suited for making inferences. In some approaches the overall factors which influence

- Inability to correctly plot the points on coordinate grid
- Inability to choose the correct scales on the axes
- Providing additional information to read graph, legends, labels etc.
- Constructing an entire graph as one single point
- Drawing iconic presentation of data
- Constructing a series of graphs, each representing one factor from the relevant data
- Conserving form of linear function in depicting non-linear data
- Difficulty in graphing slope and intercept
- Inability to construct graphs to depict situations

Table 2: Problems reported in literature on constructing graphs. Discussion about these problems is done in Chapter 2 of the thesis.

graphing are taken into consideration. The model of graph comprehension that one uses will largely determine the approach that one will take in addressing the problems raised.

The main point of focus in cognitive theories seems to be the features in graphs which help in their comprehension. When graphs are made from data, a certain 'encoding' of the data happens, and unless the readers of the graphs are able to 'decode' the graph, the graph fails in its objective. The studies on the graph perception imply that certain graphical designs are perceived more easily than others. Cleveland & McGill (1984, 1985) provide a list of the most relevant features in reading of graphs. Cleveland (1993) presents a model for understanding graphical perception and the process of visual decoding. Simkin & Hastie (1987) provide insights from Information-Processing perspective on graph perception. Kosslyn (1989) and Pinker (1990) provide a theory of graph comprehension based on perceptual and cognitive theory. Wavering (1989) classifies graph construction abilities into nine categories ranging from no attempt to make graphs to complete graph with statements about correlation. The said nine categories are correlated with Piagetian stages. Friel, Curcio & Bright (2001) identify three main components of graph comprehension going from local to global features of a graph. The perceptual organization of data can have substantial effect on the comprehension, even in case of familiar contexts and complex tasks (Shah et al., 1999). Shah & Hoeffner (2002) make use of Construction-Integration (CI) model for framework to understand graph comprehension. In this model the comprehension of graphs is dependent on three interacting factors: visual characteristics of the display, knowledge about graphs and knowledge about content. M. P. Cook (2006) presents instructional design considerations keeping in mind cognitive load, prior knowledge and working memory.

In contrast to the cognitive approach, the sociological approach does not take isolated individual as the unit of analysis. The focus is shifted from representation as a mental activity to a social activity. The relationship between a phenomena and its representation, which in the cognitive models is considered as an inherent property of the inscription, is seen as a matter of convention, and the problems learners face are due to inexperience with conventions, rather than mental deficiencies (Roth & McGinn, 1998). The emphasis that most cognitive models place on a prerequisite of the formal operational stage in the learner for construction and comprehension of graphs is questioned. The sociological approach sees graphing as a *practice* focusing on learner competence, rhetorical perspective and on affordances of graphs to collective sense making, and hence does not need to be explained in terms of cognitive deficits (Roth & McGinn, 1997; Roth & Bowen, 2000). Practice in this context refers to the actual working processes and the conventions that are followed in the domain under consideration. This practice is acquired by relevant experience and exposure to various opportunities of dealing with data. According to Bowen & Roth (1998) the interpretation of a graph according to this approach does not lie in the "understanding the representation itself as a static object but rather in understanding the social actions through which the graph was originally constructed". The emphasis is on the notion of graphing as practice. In this framework the mathematical graph related experience is linked with experience in the world (Roth & Bowen, 2001).

The review sets a framework for addressing various concerns brought out by the different approaches. Students have various difficulties in constructing and comprehending graphs. Some of these difficulties have their origin in: the design of the graphs; unfamiliarity with the domain and context in which the graphs are introduced; inexperience in inferring meaning from the features of the graph, difficulty in seeing graphs as a whole and not as a set of discreet points. For school students, textbooks are meant to provide the students with the relevant information and experiences. To understand the extent to which the textbooks support graphicacy, a detailed analysis was conducted, and is presented in the next section.

# 3 The poverty of graphicacy in NCERT Science Textbooks

Textbooks are central to the Indian education system (Kumar, 1988) and in many cases are the only source of knowledge for the students (NCERT, 2005). In India the *National Council for Educational Research and Training* (NCERT) is the highest body which publishes and prescribes the curriculum. Our sample consists of NCERT textbooks of all subjects, except languages, from Grade 5 to 10. The choice to consider all subjects reflects our belief that graphicacy is a core competence required across different subjects. We wanted to see the way graphs are used *across* the grades and *across* the subjects.

Both qualitative and quantitative techniques were used in our analysis. Quantitative techniques in the textbook analysis are mainly used in terms of space and frequency as reported in (Pingle, 1999). This can be quantification of how many times a particular word appears in the text, how much space has been allocated for a particular theme, event or topic. With these methods we are able to cover a large area, as Pingle (1999) points “[quantitative methods tell] us a great deal about where the emphasis lies, about selection criteria, but nothing [in themselves] about values and interpretation”. In qualitative research the analysis tends to be deeper in terms of structure of the textbook and affordances the unit of analysis provides to the learner. We used both quantitative and qualitative techniques which compliment each other.

## Quantitative Analysis

We wanted to get a trend of the presence of graphs in the textbooks. Therefore the analysis was conducted to address the following questions.

1. What are the different types of graphs that are present in the textbooks?
2. What is the frequency and trend of the occurrence of graphs in the textbooks, across grades and across subjects?
3. Is there any subject wise preference to presence of graphs in the textbooks?

To answer these questions the textbooks in different subjects were scrutinised for graphs. A database was created from this survey, which forms the basis for this analysis. The database is presented in **Appendix A** of the thesis. The information that we have collected during the survey is shown in Table 3.

| Parameter | Description |
|---|---|
| **Class** | The class in which the graph appears. |
| **Subject** | The subject textbook in which the graph appears. |
| **Page Number** | The page number on which the graph appears. |
| **Figure Number** | The figure number for the graph, if applicable. |
| **Legend** | The legend of the graph, if applicable. |
| **Caption** | The caption of the graph, if applicable. |
| **Graph Type** | Type of graph, namely, Line, Bar, Pie or Other. |
| **Description** | Description of the graph in the text. |
| **Data** | Whether data for the graph and its source is provided, or whether students are to collect the data. |
| **Comments** | Our comments on the design and use of the graph. |

Table 3: List of variables collected during the quantitative survey of textbooks for graphs.

| | *Subject* | | | | *Graph Type* | | | |
|---|---|---|---|---|---|---|---|---|
| **Class** | Science | Social | Maths | **Total** | Line | Bar | Pie | Other |
| 5 | 0 | 1 | 5 | **6** | 1 | 3 | 2 | 0 |
| 6 | 0 | 1 | 42 | **43** | 23 | 9 | 1 | 10 |
| 7 | 8 | 4 | 19 | **31** | 19 | 9 | 1 | 2 |
| 8 | 3 | 3 | 39 | **45** | 22 | 11 | 11 | 1 |
| 9 | 14 | 11 | 41 | **66** | 47 | 9 | 4 | 6 |
| 10 | 4 | 21 | 35 | **60** | 23 | 8 | 13 | 16 |
| **Total** | 29 | 41 | 181 | **251** | 135 | 49 | 32 | 35 |

Table 4: Table showing total number and type of graphs in the textbook in different classes and subjects. The data in this table is used to plot Figure 1.

We have categorised the subjects in three major groups, Science, Mathematics and the Social Sciences. The Social Science group includes Geography, Environmental Science, Political Science and Sociology in the grades 8 and above. Table 4 shows the number and the type graphs in each class and in each subject. We calculated the frequency with which graphs appear in each of these subjects, across all the grades under consideration. In Figure 1 the variation of the total number of graphs is shown with the grades. In Figure 1 (a) , the top left graph shows the number of graphs in different grades. The increase in grade 6 is due to the number-line graphs, while in case of grade 9 graphs on the topic of motion and Cartesian coordinate system are introduced.

Figure 1 (b) shows the different types of graphs in the textbooks. We see that the line

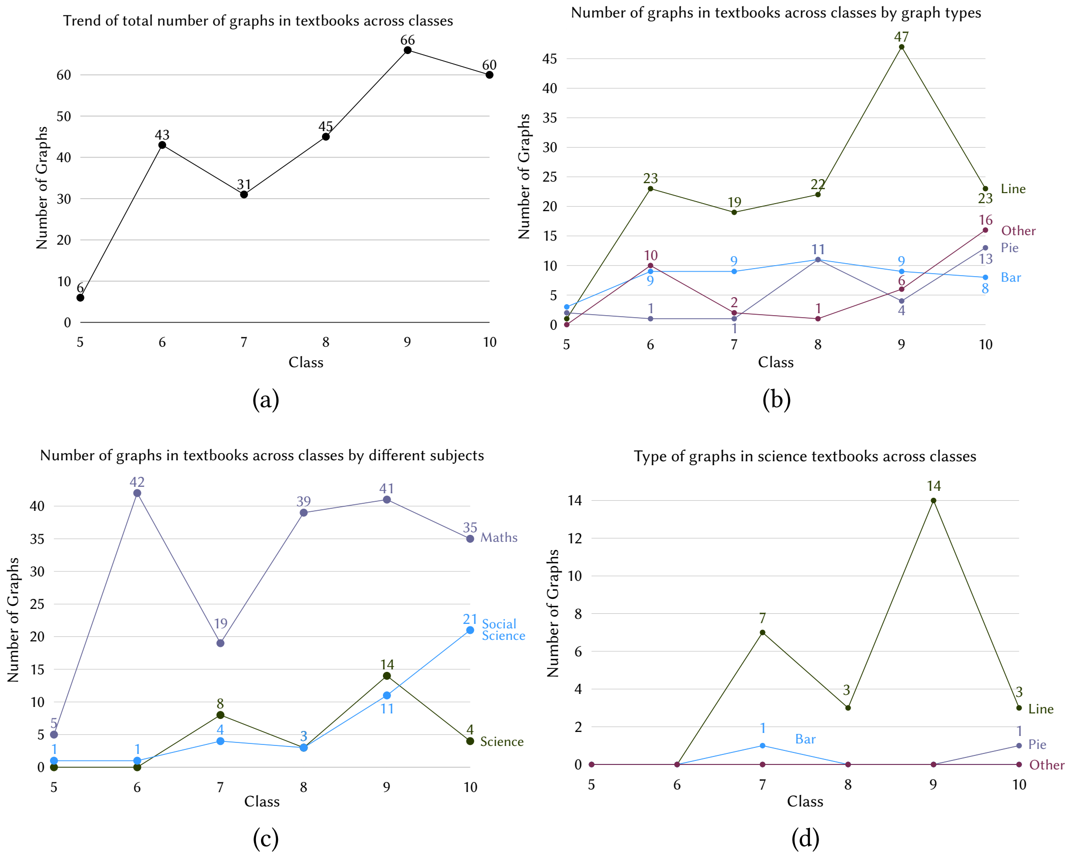


Figure 1: Variation in of graphs and their types with different textbooks. The data for this graph can be seen in Table 4 and is also presented in the Appendix. (a) The graph shows total number of graphs in all textbooks for for each grade. (b) The graph shows the different type of graphs and their variation across grades. (c) The graph shows the distribution of graphs in different subjects across grades. (d) The graph shows the different types of graphs in science textbooks across grades.

graphs have a larger representation in most cases. The category of ‘Other’ that is shown includes graphs that cannot be clearly classified into line, bar or pie types. By a *line graph* we mean a Cartesian graph. When a bar graph and a line graph are simultaneously present in a figure we have included them in the line graph category. In Figure 1 (c) the graph shows the number of graphs in different subjects across the grades. Mathematics has the largest share, with topics of number-line and Cartesian coordinate system contributing significantly to this. This is followed by the Social science textbooks, which include many statistical graphs. Finally, the science textbooks have the least numbers Figure 1 (d). A similar analysis was done for other subject categories.

The total number of graphs is the least in case of science, totalling to 29, whereas for social studies and mathematics it is 41 and 181 respectively. This is one clear indication that the school science textbooks need content which would address the issue of graphs. Out of these 29 graphs 27 are line graphs, and the other two are bar graphs and pie chart. Thus we see a trend that science textbooks mostly use line graphs, even if used sparingly. We see that in the science textbooks maximum graphs are in the context of motion.

One would expect that the total number of graphs in the textbooks would increase with the grades, that is, higher grades will have more number of graphs. The reason for such an expectation is that as the students progress through the classes they would require more opportunities to explore and engage with graphs. Just as in case of verbal literacy, complex and increasing amount of text is provided. This is also an indication that graphicacy *per se* is not seen as an important skill in the curriculum. There is no explicit or systemic planning we could find to build graphicacy across grades or subjects. Another issues is that mere presence of graphs in the textbook is not justified unless it is appropriately related to the subject matter and fulfils the goal for which it was introduced. The overall integration with the narrative, contexts in which the graphs appear and the design of the graphs, are crucial to maximising the impact of graphs in the textbooks. The qualitative component of the analysis in the next section inquires how these issues are addressed in the textbooks.

This part of the work was presented as a paper titled “**An Analysis of graphs in school textbooks**” in epiSTEME 4 Conference, in Mumbai (Dhakulkar & Nagarjuna, 2011)

## Qualitative Analysis

In this section we discuss the qualitative analysis of the graphs in NCERT Science textbooks. Table 5 contains the parameters used in critically examining the nature of graphs and the learning they support. These parameters were mostly arrived at by using framework (Roth et al., 2005) used in *Critical Graphicacy.* The parameter of *close-to-life* derives from many studies which claim that the context of the graph should be meaningful and familiar to the learners, for example, Ainley (1995). The category of the design of the graphs is crucial for graph comprehension, (Tufte, 2001; Wainer, 2007). Each of the graph that appears in the science textbooks is studied in the context of these parameters. The parameters in Table 5 give a clearer picture about the usage of graphs in the science textbooks. We analysed each of the graph in the science textbooks with respect to the parameters listed in Table 5.

| Category | Description |
|---|---|
| **Function** | What function does the graph serve in the textbook? Whether it is narrative, classificational, analytical and metaphorical representations. |
| **Reference** | Whether the graphs are referred to in the text? If they are, what is the way in which they are referred? |
| **Integration** | How well are they integrated with the overall text? How do they go with the flow of the narrative. |
| **Data Used** | What is the data used in making the graphs? Is the source of data provided? Is real data used in making the graphs. |
| **Legend and Axes** | Is the graph with key and labels to the axes? Are the variables on the axes with units and labels? |
| **Close-to-life** | Does the graph link to any everyday experience of the students? |
| **Design aspects** | Is the graph well designed? Does it have unnecessary decorative elements? |

Table 5: The parameters used for qualitative analysis of Science textbooks and their description.

The major findings from this analysis are listed below:

1. In general the presence of graphs in the textbooks is peripheral. By this we mean the integration of graphs with the main narrative of the textbook was loose and absent at times.
2. The graphing skills are not built upon as the grades progress. Connections with regards to what was learned earlier is not made explicit.

③ Only a few probing questions were asked which could lead the students to analyse the graph critically.

④ The graphs with real-world data which students could collect were few. There is a lot of potential in some of the activities for students to collect and analyse real-world data.

⑤ For most of the graphs neither the source nor the method of data acquisition was provided. This is in contrast to the scientific practice of doing so.

⑥ Some exceptions include graphs on motion, where students were actually told how the graph was constructed from a table of values for an object.

⑦ Axes in almost all graphs were labelled. In some graphs the units on axes were not given.

⑧ Readability of many graphs can be improved by redesigning them. We have shown how simple redesigning makes the graphs more readable and integrated with the text.

⑨ We could find only *one* exemplary graph which had potential for critical analysis. In terms of comprehension categorisation, most of the tasks fall in the elementary category.

Our findings addressing the second research question *What kind of opportunities do Indian school science textbooks offer to learners to engage with graphs meaningfully?* are as under:

We not only found that the graphs are sparse, particularly in science, but the nature of most graphical activities does not engage or serve the purpose of achieving the aims of graphicacy. There is no effort to build on graphicacy across grades in science. For example, there is no reference to graphs appearing in earlier textbooks. Graphs are presented as isolated entities with elementary objectives, for example to merely display data. The intermediate and advanced level questions are sparingly asked. For example, questions about extrapolating and interpolating data, predicting, inferring are rare. Exemplary examples of graph usage are non-existent. For construction of graphs too the opportunities are limited. Rarely are students asked to create graphs to answer questions at intermediate or advanced stages. This would include forming conjectures, designing and constructing experiments, collecting data to solve problems and generate answers.

We see that there exists a tremendous opportunity to explore and utilise graphs as means of teaching concepts in the sample of textbooks that we have studied. Mere presence of graphs in the textbook is not justified unless it is appropriately related to the subject matter. Graphs are best understood in a context when learners collect "real world" data and use graphs to analyse this data. In this way one can introduce some aspects of critical graphicacy in the classroom. The apparent lack of opportunities for the learners to become *graphicate*, in textbooks and curricula and ways to address them is the central theme of our work.

As a proposed remedy to the perceived lack of relevant graphical experiences in the textbook we address the research questions ③ and ④. This forms part of our next chapter in which we propose a design framework for developing activities which will make connections with various concepts through graphs.

# Part 2: Learning Contexts: Design, Development and Testing

In Chapter 4 we design a framework to address the problems raised in Part 1. In Chapter 5, we present details of the activities and field studies based on this design framework. In the last chapter we discuss major outcomes and implications of the work (Chapter 6).

## 4 Learning contexts: Design and Development

In Part 1 we have shown that one of the major problem with graphs in the textbooks is the lack of opportunities to engage with real-world data. Here we design tailored activities and test them for their efficacy in addressing these deficits. The activities are specifically designed to necessitate and encourage collection and presentation of data, to make conjectures based on a reading of the graphical representation and to test the conjectures by designing appropriate experiments. In some cases the experiments had to be repeated with new insights, leading to an iterative process.

Though the activities are varied in terms of contexts that they appear in, we have applied a common framework. The sequence of activities goes from very concrete as in the case of the mustard seed measurement to highly abstract representation of voltages across coils. Each of the activity presents a different challenge in terms of the concepts involved and the abstraction. The activities presented here in general increase in complexity, abstract nature, in the number of data points, graphicacy and reasoning skills. One of the aims is to provide the learners with familiar contexts and purposeful tasks to inculcate skills which are required in both science and mathematics, for example (Gallagher, 1979). The development and field testing of the first two activities were iteratively carried out over 3 years (2011-2013) at the Muktangan Vidnyan Shodhika, IUCAA, Pune with Grade 8 and 9 students, with (about 120 students each year). An exception to this is the electromagnetic induction activity, which was a case study conducted with two students. A pre-test and a followup post-test was administered to the students covering concepts from astronomy, physics and logical reasoning. We present here a rationale for the design framework and the process/work flow of the activities. This chapter addresses the research question (3).

## Implications for the design from the literature

Main findings and implications from the literature regarding development of graphicacy are summarised below. These implications and our own observations from the analysis of the textbooks informed the development of activities.

We use the notion of context as explicated by (Janvier & Bednarz, 1989). A context here is seen equivalent to a *situation*. It is the concrete basis from which many abstract mathematical ideas are derived from the real world. Contextualisation lies in the intersection between the phenomena (real-world, concrete situations) and equations (abstracted mathematical formalism), as shown in Figure 2. The idea of contextualisation cannot be separated from problem solving and is distinct from application of mathematics. Contextualisation in this sense provides an operational space for learners to make the connections between the abstract and concrete embedded in solving a problem.

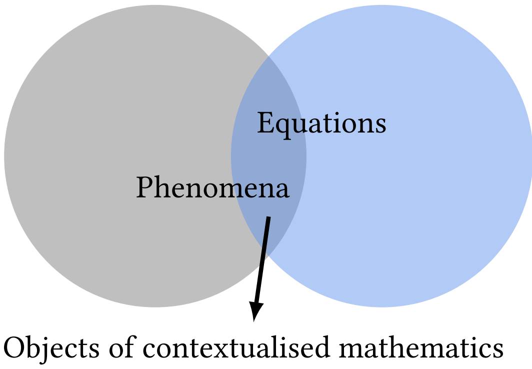


Figure 2: Contextualisation is the space positioned between the real-world phenomena and abstracted mathematical ideas, after (1989).

The problems that we give to the learners will be successful in achieving their potential if they are in a familiar context, and have concrete goals which build on their prior knowledge (both *practice* and *content*). For example, learners might know how to do things, like measuring with a scale, or they might know about certain facts or concepts. Making use of the prior knowledge during discussions was found to be very useful. This makes the learners contribute to solve the problem, thus making it a personal endeavour for them.The mathematical transfer of learning in case of a real world problem is different than in a contextual word problem depicting a realistic situation (Roth, 1996).

Thus we need to design learning situations which are set in *familiar* (close-to-life) contexts of the learners, and tasks which have *understandable problems* (what is to be done?). Once the problem is recognised, the discussions on how to solve the problem (*brainstorming*) gives the learners chance to design and construct conjectures and experiments. During the discussion about the problem, inputs from the learners are an important component of the discourse. Conjectures about solving the problem are made, refuted, modified and accepted. This process includes setting a dialogue with the learners, taking inputs from prior knowledge and reasoning, and finally using these to solve the problem.

Once it is decided how to solve the problem, the next step is to make constructions for conducting experiments and observations. This step makes things concrete. The construction involves both working alone and working in groups. In the group working scenario, the work is divided among the learners and eventually the roles are interchanged to ensure everyone participates in all activities. For collecting data, concrete parameters were changed. At each stage the learners were aware of why they were doing something. The data was recorded in a tabular form during the experimentation. The possible sources of error were noted down, along with the precautions to be taken while performing the experiment. The reports were written so that others reading them would be able to repeat the experiments, just like in case of scientific practice.

The experimental construction led to *data generation* which was represented in multiple ways (*graphical, algebraic, tabular, verbal* among others). This led to construction of mathematical models, with correlations between the experimental parameters. The verbal description of the data was used to describe the collected data. The graphs allowed to make inferences and predictions, thus testing conjectures in the process. The activities were designed so that each experiment or observation is grounded in something concrete and tangible (for example, the number of turns in a coil) and thus the resulting graph could be linked to the phenomena. This made the connection between the abstract features on the graph and the concrete things they represent strong. In many cases the flow of the process was both ways: to and fro between the concepts. This is indicated by the double sided arrows in the diagram. The analysis based on data can lead to inferences, conclusion and solutions to the conjectures and problems. A *public display* in form of classroom presentations and discussions of these results, led to further discussion about the data, models, conjectures and problem itself. In some cases the public display led to revision of the earlier ideas and conjectures and the process became iterative. The core ideas of the designed activities and their work-flow are shown in Figure 3. The flow of the activity can be uni-directional ( as noted by $\longrightarrow$) or bi-directional (as noted by $\longleftrightarrow$). These concepts, when they are relevant in the activities are shown in different formatting as: `(concept)`.

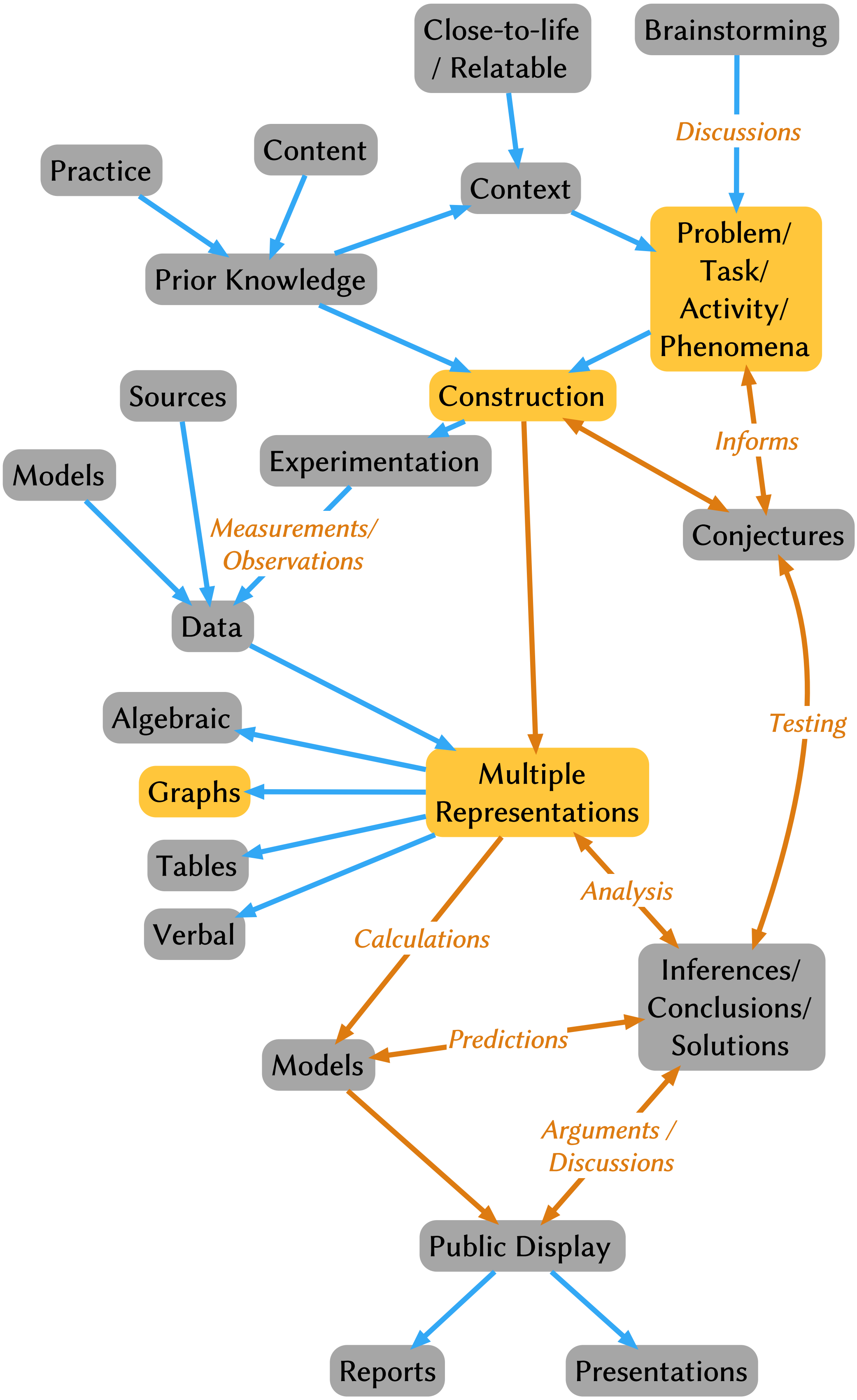


Figure 3: Schematic diagram showing core ideas and the work-flow of the activities.

Within this framework, collaboration and public (peer group) display and discussion of the artefacts and inscriptions provided the social space for the unfolding of the activities. Tangible form of the products (like constructing objects, reports, graphs) made public display and discussion possible. This idea is derived from *constructionism* as set forth by Seymour Papert (Papert & Harel, 1991; Papert, 1980) and has parallels in the *studio based* approach to education (Hetland, Winner, Veenema, Sheridan & Perkins, 2007). Because of its greater focus on learning through making, Papert's approach helps us understand how ideas get formed and transformed when expressed through different media, when actualised in particular contexts, and when worked out by individual minds (Ackermann, 2001). The construction context is one of the essentials aspects of the activities. It allows learners to navigate between the abstract and concrete levels of situation in focus. It has been shown that in case of scientists interpreting graphs the movement between abstract and concrete is not just one after the other, but appears to be simultaneously from concrete to abstract and from abstract to concrete (Roth & Hwang, 2006). Also the activities are designed in such a way that they can be done by learners of different grades with different goals and learning outcomes. That is to say, the same activity can be repeated with a new set of skills and knowledge to give a new learning outcome through the activity.

**Data Collection and Handling**: Real world data collection and handling even for simple tasks can be a rich experience for learners (Curcio, 1987; Wavering, 1989). Classifying data and representing it in various forms can lead to a deeper understanding of meaning of data (Pereira-Mendoza, 1995; Hutchison, Ellsworth & Yovich, 2000). The ability to move between different representations of the same data is not easy. When this same data is used for further analysis, to find mathematical patterns, to build a model, to predict, to answer questions, to verbalise, the graphs could be used for display and as rhetorical devices. During the course of such an activity many associated concepts from mathematics and science like probability, statistical measure, experimental error etc. could also be introduced in a contextualised manner (Lehrer & Romberg, 1996). Also the familiarity with the data collected by students themselves could lead to successful learning of many fundamental features of graphing (Åberg-Bengtsson, 2006). Such contextualised experiences could help students to build and expand on their repertoiré of skills to understand the mathematical relations expressed in graphs. This involves the task as context for calculations and measurements with the collected data.

**Graphs**: Graphs are central to the activities by design. They form the link between various concepts. The centrality of graphs and its relation to other concepts in the framework is shown in Figure 4. We analyse the graphs in the activities with the categories listed. The graphs played a crucial role in completion of the activities as they do in science. The categories are substantiated with examples in the activities.

We had both hand drawn and computer drawn graphs as part of our activities. In case of hand drawn graphs the data sets were typically small. The hand drawn graphs were part of the reports that learners submitted on the activity performed. This allowed the learner to understand these graphs in relation to the other forms of representations in the report. Though the advantages of computer drawn graphs are many, their true potential comes when the data sets are large. (Mokros & Tinker, 1987) and (Barton, 1998) argue that using computers to draw graphs gives many advantages to young learners. The graphing on computers allows for multiple modalities of learning, providing a *real-time* and mathematical link between a concrete experience and its symbolic representation (Pratt, 1995, 3). Particularly the reduction in cognitive load and time saved from "drudgery" of graph drawing can be used for "What if?" type discussion questions. The immediate feedback that the learners get while changing the parameters of the graph such as scale or order, helps immensely in developing the skill of reading a graph and its meaning (Wavering, 1989). This also indicates that traditional approaches to learning need to be reconsidered in a computer rich learning environment (Arcavi & Hadas, 2000). Activities with *real-time* data collection and display on the computers have resulted in significant improvement in learning (Adams & Shrum, 1990). This approach allows learners to participate in the process of learning similar to that of a scientist working in a laboratory, trying to understand complex factors influencing observations (Nachmias & Linn, 1987). A schematic diagram showing effect of graphs on various components and process of the designed activities is shown in Figure 4. Each of the activities was analysed keeping these issues in focus.

In designing of the activities we used the following technological tools, namely expEYES for collecting and storing the data, and GeoGebra for dynamic mathematics. The use of these tools are exemplified in the activities and addresses one part of the research question (4). The rationale for selection of the appropriate tools is discussed. Each of the activities that we present has multiple learning objectives spanning different areas of mathematics, science and statistics. In the following section the problem statement of the activities, the work-flow and highlights are summarised. The learner response for each activity was analysed in detail for processes and products and is presented in the corresponding chapter in the thesis.

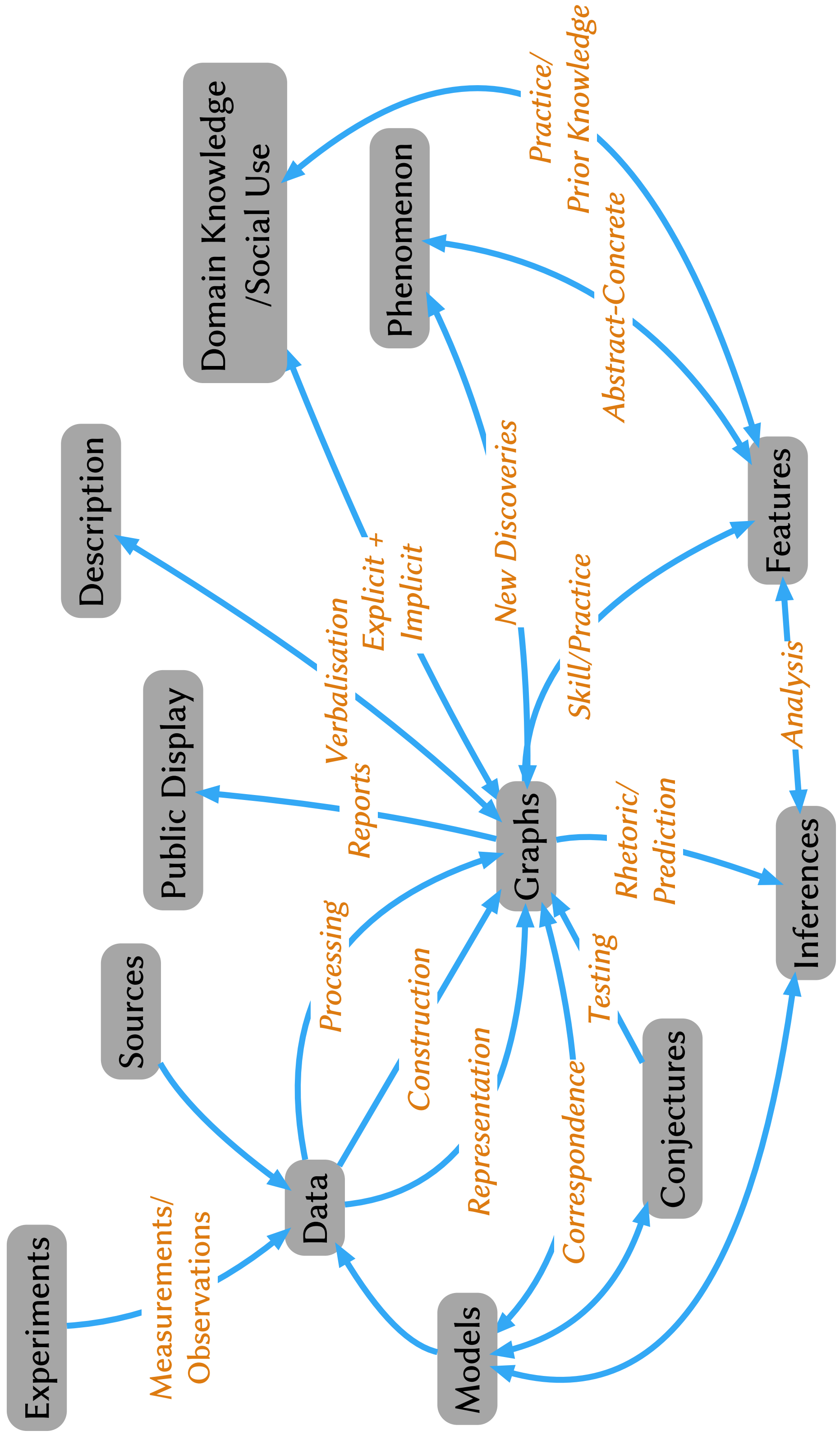


Figure 4: Schematic diagram showing major relations of graphs to other components and process in the activities.

# 5 Activities

The three activities described below address research question (4). In what follows we describe the activities in brief and major outcomes of these activities from the perspective of student learning. The activities themselves involve construction and data collection in a problem solving context. Each of the activities establishes a real world connect to mathematical concepts in familiar contexts. The first two activities have technology use at crucial points during the activity, while the third activity is wholly dependent on the technology used. In each of the task the students gave varied responses, and particularly for the first two tasks not all students were able to successfully complete the tasks.

## 5.1 Measuring the Mustard Seed

In this activity the students measure the average diameter of the mustard seeds. Mustard seeds are a very common ingredient in the Indian cuisine and are found in almost all households in India. There are two varieties of mustard commonly found in the markets, one variety has seeds almost double size ($\sim 2$ mm) of the other one ($\sim 1$ mm). The size of the seeds is just right enough to make measurements possible with the help of a ruler, also they being almost spherical and their easy availability makes them ideal for such experiments. The task can be seen as a first step towards mathematically modelling more challenging problems from real life situations which have little scope in the standard school textbooks.

The activity was carried out over three years (2011-13), with different set of students each year. This was the first activity in the set, giving them idea of scale and measurement. The subsequent activities built on this idea, of using indirect measurements and using mathematical models to solve the problem at hand. The data collected during the development and field testing consisted of researchers notes during the classroom discussions, photographs and reports submitted by the students.

The work flow for the activity was as follows. The applicable concepts from the framework in Figure 3 are shown as `(concept)`:

(1) Discussing estimating size of everyday objects. `(prior knowledge)`

(2) Discussing the direct and indirect methods of measuring `(prior knowledge)` `(context)` the mustard seed diameter `(problem)`.

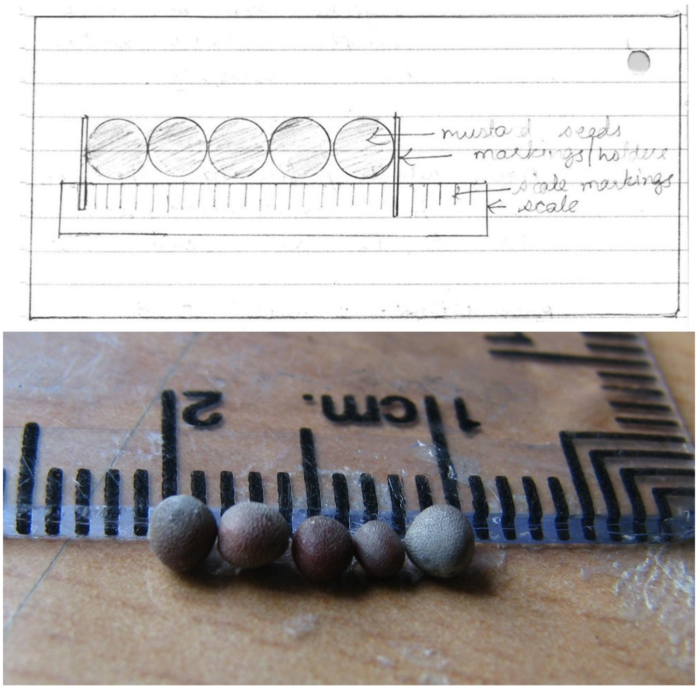


Figure 5: Diagram and a photo from the mustard seed task depicting the placement of 5 seeds near a ruler to measure their combined length.

3. Measuring the length aligned mustard seeds in varying numbers and recording this. `(data) (conjectures) (experimentation)`

4. Discussing possible sources of error while making the measurements.

5. Performing simple statistical analysis on the data collected

6. Collecting data in the form of a table and plotting the data on a graph. `(construction) (multiple representations)`

7. Making a mathematical model from the data that was collected, making connections of the mathematical model to observations. `(models) (inferences) (analysis)`

8. Reporting the work done, with graphs, tables and diagrams `(public display) (construction)`.

9. Collecting data from all students and plotting the graph using GeoGebra. `(public display)`

10. Classroom discussion about the the reports, and the combined graph. `(inferences) (conclusions)`.

The students were familiar with mustard seeds and the properties of a circle (like area and circumference) and a sphere (volume), direct proportion and linear equations. The classroom discussions started with idea of indirect measurements, with a few examples from the textbooks. The students were asked to guess the size of the mustard seeds, after allowing them a close examination of these seeds. Students typically answered from few centimetres to few millimetres. When they were asked "How can we measure the diameter of the mustard seed?", some of the students came up with ingenious answers, while some gave simple answers. Some of the students came up with some ingenious methods of measuring the diameter. One of the students suggested that a thread should be wound on the seed, and then the length of the thread can be measured easily with help of a ruler. Another student suggested an even more elaborate method: we can find out the volume of displacement of water due to one seed and then from the volume of the water displaced we can find out the volume of the mustard seed and from this volume we can find out the radius and hence the diameter. But in most of the answers the idea that we have to measure only *one* seed was present. This idea was questioned by asking students to look at the mustard seeds, they noted that all seeds are not of the same size. Hence the idea of measuring an *average* of the seeds was established. This way the idea of doing multiple measurements and taking averages was brought in. Then the discussion led to using the task of measuring the diameter of the mustard seed with help of a ruler (1 mm as the least count).

The measurement involved aligning the mustard seeds along a ruler and measuring the length covered by them. Then the students were to find out the average diameter for each of the set of seeds. The students were asked to submit a written report on this task. The report contains detailed description of how the students conducted the experiments and their results. The representation of the data using graphs always has a concrete connection for each of the point. The crucial point in the activity arises when the data from the entire class was collected and plotted. Since the mustard seeds come in two sizes, we get two distinct plots corresponding to the two sizes. In the classroom discussion that followed this point was inferred. The dynamic mathematics tool here allowed us to plot the generic linear equation $y = m \times x$, with variable slope, along with actual data points. The variation of the slope $m$ in the model can be directly linked to the size of the seeds. After the combined plotting, an algebraic model was derived from the data through discussions. The students further elaborated on this new information in their reports.

In this activity an explicit connection between the mathematical knowledge about averages, direct proportion and linear equations was made to solve a problem at hand. As suggested by the textbook analysis these connections are usually not made. The idea of independent and dependent variables, errors were embedded in the activity in an organic way. This activity could be seen as a first step in mathematical modelling and making use of graphs to understand and explore the model in context of solving a problem. As evident from their reports many students could think concretely about the slope of the graph as the average diameter of mustard seed.

This work was first presented as a paper titled "**Measuring the mustard seed: A first exercise in mathematical modelling**" in epiSTEME 5 Conference in Mumbai (Dhakulkar, Dhurde & Nagarjuna, 2013) conference and subsequently published as an article titled "**Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling**" in *School Science Review* (Dhakulkar, Dhurde & Nagarjuna, 2015).

## 5.2 Measuring the distance to the Sun

In this task the students estimate the ratio of diameter of the Sun to its distance from Earth. The method provides an order of magnitude estimate of the ratio by constructing a pinhole camera. In contrast to the mustard seed task, a result from properties of similar triangles is used to make a mathematical model. The actual measurements require students to construct the experimental setup, shown schematically in Figure 6. The activity was carried out over three years with the same set of students in the previous mustard seed activity.

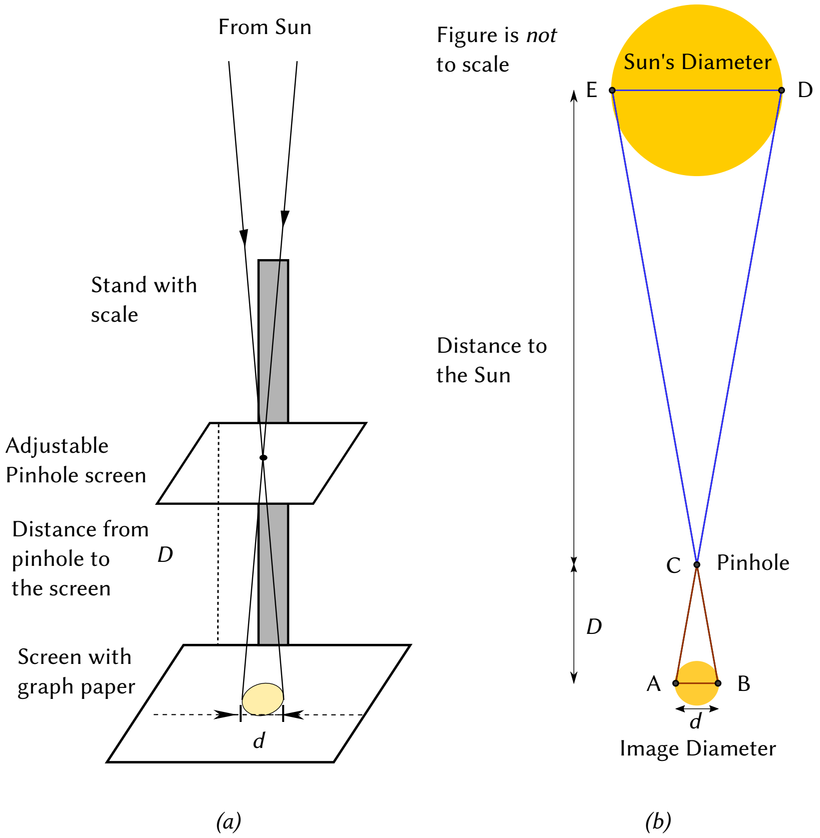


Figure 6: Schematic illustration for the setup of Sun distance/diameter task. The experiments consists of varying the height $D$ to get different values of the diameter of the image $d$. Figure is not to scale.

The work-flow for the activity was as follows.

① A pre-test question about how to find distance from the Earth to the Sun `(collaboration) (prior knowledge)`.

② Classroom discussions on the student responses to the question above.

3. Classroom discussion using GeoGebra on similar triangles and their properties and how they can be used to solve the current problem. `(construction)` `(models)` `(multiple representations)`

4. Creating a 'mathematical model' to depict the situation and identifying the main components. `(conjecture)` `(model)`

5. Demonstration and working principle of a pinhole camera. `(construction)` `(experimentation)`

6. Discussions on possible errors, precautions to be taken while performing the measurements. `(experimentation)` `(brainstorming)`

7. Construction of the pinhole camera and assembly for the experiment. `(construction)`

8. Measuring the values of diameter for different heights on the scale. `(data)`

9. Analysing the observations and estimating distance to the Sun or its diameter by using numerical calculations and by using the slope of the graph. `(analysis)` `(models)` `(inferences)`

10. Writing a report including showing the required estimate graphs, tables, description of the experiment.
`(public display)` `( multiple representations)`

The Sun measurement task was more involved than the mustard seed task. The varied answers given by students to the question "How can we measure the distance to the Sun?" lead to rich discussions in the classroom. For example, some students used the idea of **speed = distance × time**, knowing the speed of light and time light takes to reach us from the Sun, the distance can be found out. But when such responses were probed a bit deeper, by asking how we know it takes about 8 minutes for the light to reach us from the Sun, the students could not answer. Some students, used a "rocket ship" to find out the distance, just like we can find the terrestrial distance using a car. The answers themselves gave insights to the way students think. The role of prior knowledge in their attempts at problem solving became clearer. The students did not associate geometry they had learned to the act of measurement, implying that knowledge transfer across subjects does not happen easily. Our textbook analysis suggested that this connection between mathematics and other subjects was lacking. Making this connection was explicitly carried out in this activity. The researcher used prior knowledge of the learners to create the mathematical model for the experiment. This involved discussions around the idea of similar triangles and pinhole cameras using dynamic mathematics software. During the discussion the similarities between the pinhole camera and similar triangles were brought to notice. The reasoning continued to finally create a simulation of the experimental setup with the help of the students using the dynamic mathematics tool. One of the important outcomes of this part was to enable students to measure something that they knew only as a fact. The textbook only provides this information as a matter of fact, not giving any sources or methods by which this could be obtained.

The next step involved the students building the experimental setup required to make the measurements. Students formed groups and were guided by mentors while constructing the pinhole cameras and their mounts. The making of the pinhole cameras and assembly of the experimental setup involved many design decisions. The assumptions and precautions to be taken during the activity were discussed with the students. The discussions also involved various types of errors that students might face during the experiment. The experiment was a group task as it required more than one person to complete it. Each student in the group had a chance to perform all aspects of the experiment. The change in parameters could be concretely connected to the graph.

The reports of the students contain a detailed account of the activity. These reports contain the mathematical model formed and the results of the activity. The data was represented in a table, in the form of an equation and a graph. The connection between various representations that the students are making can be seen in these reports. The slope of the graph in this case is concretely connected to the problem. This result is verified by algebraic means also. This activity establishes a rich learning experience for the students, addressing in part research question ④. The use of technology is important but is limited in this case.

## 5.3 Exploring Electromagnetic Induction

In this task learners explored the interaction between electric current and magnets by constructing coils of different parameters. The learners were given a data logger expEYES (http://expeyes.in) to measure the output voltage of the coils. This task was conducted as a case study with two students, instead of an entire class. The use of a computerised tool becomes a necessity in this case as the phenomenon is a transient one which lasts only for a few hundred microseconds and cannot be recorded with a regular multimeter. The context of construction of coil here provides a concrete grounding for understanding the phenomenon.

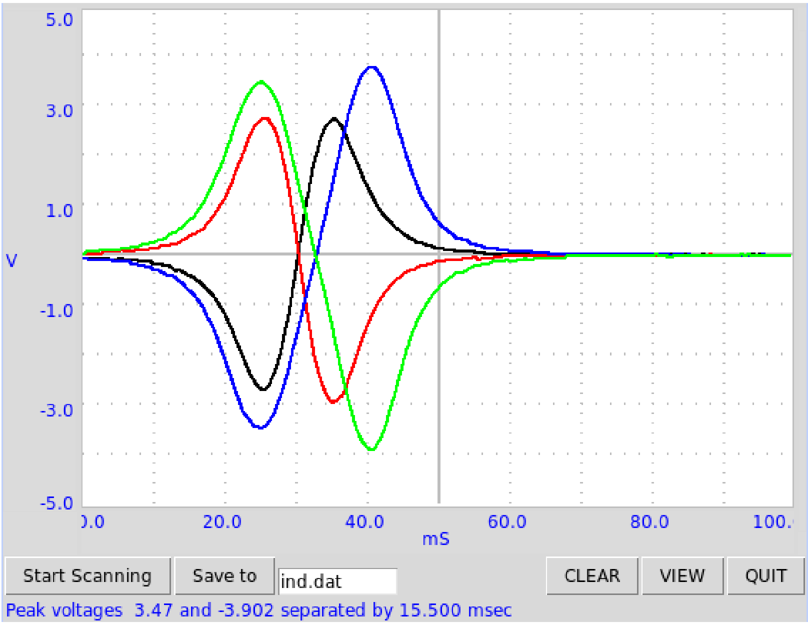


Figure 7: A result from the experiment in Electromagnetic induction (EMI) activity. The graph shows the voltage drop across a coil when a magnet is passed through the coil. Each colour in the graph presents different observation with changes in experimental parameters. Note (a) the change in polarity and amplitude for different waveforms (b) the complete event takes place in about 70 milliseconds (c) second peak is always larger in magnitude than the first one.

The work flow for the activity was as follows.

1. Discussion about the nature of electromagnetic induction and parameters it depends on. `(prior knowledge)` `(close-to-life)`
2. Defining the problem and defining experimental parameters. `(brainstorming)` `(construction)` `(conjectures)`

3. Constructing induction coils from wires with various parameters as decided above. `(construction) (experimentation)`

4. Taking observations by varying different parameters. `(experimentation) (data) (multiple representations)`

5. Analysing the data collected and testing the conjectures. `(testing) (conjectures) (models) (inferences) (analysis) (multiple representations) (rhetoric)`

6. Writing a report explaining the process of the experimental procedure, the conjectures and inferences. Presenting the work in front of the peers. `(public display) (discussions)`

The activity was situated in the everyday context of electric motors which run on the principle of electro-magnetic induction (EMI). The students had also learned factual information about the phenomenon through their textbooks. They knew factually of various manifestations of the EMI, for example, reversing of deflection of magnetic needle when polarity of the magnet is changed. They were also aware of the various parameters that affect the EMI, for example the number of turns in the coil. The problem that was posed to them was to test and verify how these parameters affect the induced voltage. The students constantly interacted with the researcher during the duration of the activity over a week. The activity was an open ended exploration with experiments designed and conducted by students. Interactions with the students were video recorded for analysis.

The experiments involved passing of a magnet through a coil and recording the voltage induced in the coil. The experiments involved construction of the coils and of the experimental setup. In other similar studies, students usually do not construct the coils to be used in the experiments, for example, see (Kingman, Rowland & Popescu, 2002; Amrani & Paradis, 2005; Bonanno, Bozzo, Camarca & Sapia, 2011). The passing of the magnet through the coil is a transient phenomenon ($\sim 100$ ms) and for capturing the voltage generated, a data capturing device was used. Since the number of data points ranged typically from few hundred to few thousand, plotting the data with a computer became imperative. Multiple readings created a large amount of data for each setting of a parameter. The parameters that were varied included the coils, magnets and the speed of approach of the magnets. When the graphs for a given parameter were plotted together, they clearly showed the effect of change in the parameter. For example, in Figure 7 we can see the effect of changing polarity and strength of the magnets.

The explanations given by the students centred around the graphs. We could see multiple roles being played by the graphs. The graphs allowed the phenomenon to be seen. The discussions about the effect of changing various parameters could be seen readily from graphs. The graphs also allowed for testing of conjectures that were made earlier. The graphs also allowed students to discover new aspects of the phenomenon and giving possible explanations for them. For example, that there are two peaks in the induced voltage, the second peak in the sinusoidal graph is always higher than the first one. Among the conjectures made by the students, only one turned out to be incorrect. At the end of the task the students could directly argue and explain from the graphs about the phenomenon. The conceptual flow between the concrete phenomenon and the abstract graph was very well established. The reason for this perhaps was the fact that they themselves designed the experiments to test their own conjectures. This was evident during the discussions and presentation by the students.

In this activity we have converted an essentially qualitative demonstration of EMI in the textbook to a conjecture driven quantitative experiment. The transient nature of the phenomenon was captured only because of the availability of a tool. This task addresses the "technological tools" part of research question (4) in a deeper way than the previous two activities. The technological tool used in this activity, lead to easy collection and storage of data in electronic format. The tool also gave immediate feedback in a graphical format, thus enhancing the concrete-abstract connection. This provides us with the criteria that a technological tool should have in order to help learners become graphicate.

> This part of the work was presented as a poster titled "**Exploring the phenomena of electromagnetic induction**" in epiSTEME 6 Conference, in Mumbai (Dhakulkar & Nagarjuna, 2015).

# 6 Reflections and Outcomes

The field of science and mathematics education identify graphicacy an important competency required of students. In our work we seek to answer the problem of how to approach the problem of developing graphicacy as a core skill, though the existing curriculum does not pay attention to developing the skill explicitly. The textbook analysis gives the evidence for the poor state of graphicacy in the curricular context. Based on this the rest of the work provides a project-based, constructionist framework to develop activities towards developing graphicacy.

What we have attempted through the diverse set of exercises that were performed is to find common threads in these. We have tried to address the concerns from the research in science and mathematics education by providing students with opportunities to handle real world data in a construction context. The activities show the feasibility of addressing the gaps that were identified. One of the realisations is that the students really need to know that the mathematical principles and activities that they encounter in different subjects and different topics are the same. The idea of applicability of mathematical principles to other contexts should be made clear.

As Monk (2003) suggests, students need to repeatedly encounter graphs as a means of communication and of generating understanding, as the students move across the grades. But instead what one finds is that the graphical practices are spread far and wide and often do not make any reference to each other. This is especially true of the links across the subjects.

While introducing graphs to the students as a first step there is a need to introduce these in a context in which students are familiar with. The already familiar context is now seen in a new light of another representation, namely the graphical representation. When exploring the effect of one type of representation on another, a better understanding of the concept is possible. In fact studies in mathematics education research indicate that functions and graphs are one of the first places where students use one form of representation to understand another, graphical and algebraic in this case (Leinhardt et al., 1990). The ability to move between different modes of representation and understanding the meaning between them is a desirable quality that we want students to develop. The emphasis here of course is to understand what something in this representation means in another representation. For example, what physical significance does a steep line in a given context have? Why should this be limited to only these two forms? It can and should be linked to a multitude of representations and also the meaning of the graphs in each one of them should be made clear.

Each of the activities that we have presented here, come with a set of linkages to a rich repertoire of concepts, and the graphical activities in each one of them make it possible to bridge various concepts such as measurements, slope, statistical quantities etc. This goal is reiterated in curricular and vision documents across science and mathematics education. The *National Curriculum Framework 2005* indicates that the students must be empowered to collect and analyse their own data (NCERT, 2005). The National Council of Teachers of Mathematics (NCTM) in its *Principles and Standards for School Mathematics* lists graphs and data handling as essential skill to be developed across grades (NCTM, 2000). Project 2061, *Benchmarks for science literacy*, also emphasises the ability to handle data and graphs as a core competency across grades (American Association for the Advancement of Science [AAAS], 1994). The graphical activities set in different contexts provide the mechanism by which the different concepts which are spread across grades and across subjects can be related. We see the graphical activities acting like a thread which weaves through the otherwise disconnected set of concepts. We hope that introducing the students to such diverse set of activities, will help them understand graphs around them in a better way.

Just as it is expected of a person who leaves school to have skills of numeracy and literacy, we strongly recommend that graphicacy be taught and evaluated before graduation for all citizens.

## Major Outcomes and implications

The broad outcomes of this work can be divided into two major categories: one was the survey of the textbooks for the graphical practices, second was the developing and testing of the activities for developing graphicacy. These will be elaborated in the thesis and are listed as under:

(1) In this work we identify graphicacy as a neglected item in current scenario of science education - this is formed on the basis of the textbook analysis that was done. This part addresses research questions (1) and (2).

(2) Based on our experience we provide recommendations for the curriculum and textbook designers:

§ Emphasis on more integrated approach towards teaching of science and mathematics with graphicacy as an area of interaction between them.

§ Linkages between different concepts to be exposed via construction contexts with graphicacy as a tool for linking.

§ Activities instead of chapters for an organic integrated approach towards learning. Each of this activity has a special emphasis on connecting to various concepts and in various contexts across subjects and grades. Thus learning happens in the context of construction and we do not have to make connections artificially, but they develop organically.

§ The design of the textbooks and presentation of graphs needs an overhaul in terms of the quality of the graphs and their design. This is true of other graphic formats like illustrations and photographs too.

§ Students should be exposed to exemplars of good design and analysis of graphs as a first step.

§ Many graphs can be redesigned to make them effective.

§ The graphs present in the textbooks should avoid unnecessary decorative design elements.

(3) We present a design framework, for developing activities to address issues raised in research questions (1) and (2). This addresses research question (3).

(4) We propose and explore the way in which such a skill can be introduced with linkages spanning across the subjects and grades in the form of the learning contexts. These contexts are constructionist in nature, with connections to real-world data.

(5) The present work is both exploratory and developmental in terms of resources to bridge the gaps between subjects and grades.

(6) One of the major outcomes of this work is the activities which were designed. This addresses research question (4).

(7) Reporting analysis of the field studies of the developed activities. These studies confirm some of the findings of literature and gives us an insight into many of the problems that the students face and their possible solutions.

(8) Role of ICT in graphicacy: once the skill of graphing is developed, the need is to provide students with exploring the meaning in the graphs rather than

mechanical plotting of the same. And this is the part where technology helps us at different stages. Exemplars for this kind of technology are dynamic mathematics softwares, like GeoGebra and data collection devices, like expEYES and Arduino. This addresses the technological tools part in research question (4).

## Limitations

The work presented here is by no means complete. One of the limitation that we know is that the activities were developed in a summer camp and not in a real classroom setting. Since the activities were carried out by the researcher, it is not clear what competencies will be required by the teacher to conduct these in the classroom. However, we have made an attempt to write the activity chapters in the thesis to be readable by practising teachers. The use of ICT tools is implicit in almost all the activities. The presence of ICT tools opens up new avenues of handling graphs by allowing the learners to focus more on conceptual issues related to meaning rather than on mechanical issues of construction of graphs.

We faced a larger dilemma about the nature of the activities developed. One approach was to consider a couple of activities and work with a large number of learners to check the efficacy of the activities. The other approach, which we eventually followed, was to develop a framework which help design many such activities. The framework developed here is a tool for development rather than for testing efficacy of such activities. However the framework clearly lays down the important ingredients to facilitate evidence based teaching practice.

The activities developed in the framework suggested here are by no means a solution to all problems pertaining to graphicacy. The suggested activities can be seen as a start towards developing a more critical and comprehensive approach towards handling of graphs by learners. The activities are suggestive and do not cover all the aspects of the syllabus but do address some of the core issues of graphicacy.

## Further Work

For further establishing the claims made we need to further work along these lines:

1. To develop and field test the concept paper on GPS/GIS activity.
2. Analysing and reporting of the activities developed and tested but not reported in the thesis: this includes activities on exploring human senses of hearing and seeing, exploring mathematical functions, exploring AC and DC.
3. Developing and testing further localised and contextualised variants of the activities.
4. Studying the effects of re-designing graphs in the textbooks, as suggested in the work, on student learning.
5. Studying the retention effects of the activities? How well do the skills learned in the activities transfer to other domains and contexts.
6. Studying how the activities fare in a real classroom setting.
7. Studying to what extent the graphicacy skills transfer to everyday domains, like reading newspapers.
8. Studying how the graphical competency maps to required competencies of science and mathematics.
9. Studying if there exist any contextual differences in the graphical understanding in science and mathematics learning.
10. Focussing on the possibility of graphicacy being taken as a concrete and comprehensive measurable outcome for science literacy.

# Appendices

**Appendix A: Data collected for textbook analysis**

*Appendix A* has data used in the textbook analysis in a tabular form. The parameters mentioned in Table 3 in Section 3 are detailed out. This is a mapping of graphs in the entire sample of textbooks.

**Appendix B: A brief history of graphs**

*Appendix B* discusses the historical origins of graphs for displaying data. We discuss the contributions by Johann Lambert and William Playfair in pioneering data display by using graphs. It is evident that though the conceptual prerequisites for constructing graphs were present by the middle of eighteenth century, the graphs did not appear till almost end of the century. Only after the work of Playfair, we see that the idea of the graphical method for displaying data caught on. We see that it is not easy to grasp the idea of graphical method even when one has the conceptual tools needed available.

**Appendix C: Activities not analysed in details**

*Appendix C* contains a set of activities that were developed and tested but are not a part of the thesis. This includes a concept paper on using GPS data to learn motion (Dhakulkar & Nagarjuna, 2011b), use of dynamic mathematics software for teaching history of science (2011a).

# List of Publications

The following articles were published during the course of this study.

1. Dhakulkar, A. & Nagarjuna, G. (2011). An Analysis of Graphs in School Textbooks. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 127–131). Macmillan

   This article is based on the quantitative part of the textbook analysis presented in Chapter 3 of the thesis.

2. Dhakulkar, A. & Nagarjuna, G. (G.). (2011a). Epicyclical Astronomy: A Case for GeoGebra. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 324–328). Macmillan

   This article is based on the construction of epicyclical astronomical models used for teaching history of science and was presented as a poster in epiSTEME 4 Conference in Mumbai.

3. Dhakulkar, A. & Nagarjuna, G. (G.). (2011b). From Geography to physics: How does geography help students learn motion? In S. Hellmann, P. Frischmuth, S. Auer & D. Dietrich (Eds.), *Proceedings of the 6th Open Knowledge Conference, OKCon 2011, Berlin, Germany, June 30 & July 1, 2011.* (Vol. 739). CEUR Workshop Proceedings. CEUR-WS.org

   This is the concept paper presented for developing activities around GPS data for teaching basic concepts of motion.

4. Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2013). Measuring the mustard seed: A first exercise in mathematical modelling. In A. Jamakhandi, E. M. Sam & G. Nagarjuna (Eds.), *Proceedings of epiSTEME 5: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 213–219). CinnamonTeal Publishing

   This article is based on the field studies of the mustard seed activity with the batches from first two years.

5. Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2015). Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling. *School Science Review*, *96*(356), 63–68

   This article is based on the field studies of the mustard seed activity for all the three years.

6. Dhakulkar, A. & Nagarjuna, G. (2015). Exploring the phenomena of electromagnetic induction. In S. Chandrasekharan, S. Murthy, G. Banerjee & A. Muralidhar (Eds.), *Proceedings of epiSTEME 6: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 276–284). CinnamonTeal Publishing

   This article is based on the case study of electromagnetic induction.

# References


Åberg-Bengtsson, L. (2006). ‘‘then you can take half …almost”—elementary students learning bar graphs and pie charts in a computer-based context. *The Journal of Mathematical Behavior*, *25*, 116–135.

Ackermann, E. (2001). Piaget’s Constructivism, Papert’s Constructionism: What’s the difference? In *Constructivism: Uses And Perspectives In Education, Volumes 1 & 2). Conference Proceedings, Geneva: Research Center In Education/ Cahier 8 / September 01. PP* (pp. 85–94).

Adams, D. D. & Shrum, J. W. (1990). The effects of microcomputer-based laboratory exercises on the acquisition of line graph construction and interpretation skills by high school biology students. *Journal of Research in Science Teaching*, *27*. doi:10.1002/tea.3660270807

Ainley, J. (1995). Re-viewing graphing: Traditional and intuitive approaches. *For the Learning of Mathematics*, *15*, 10–16. Retrieved from http://www.jstor.org/stable/40248173

Aldrich, F. K. & Sheppard, L. (2000). ‘Graphicacy’: The fourth ‘R’? *Primary Science Review*, *64*, 8–11.

American Association for the Advancement of Science. (1994). *Benchmarks for science literacy*. Oxford University Press.

Amrani, D. & Paradis, P. (2005). Faraday’s law of induction gets free-falling magnet treatment. *Physics education*, *40*, 313.

Anscombe, F. J. (1973). Graphs in statistical analysis. *The American Statistician*, *27*, 17–21. doi:10.1080/00031305.1973.10478966. eprint: http://amstat.tandfonline.com/doi/pdf/10.1080/00031305.1973.10478966

Arcavi, A. & Hadas, N. (2000). Computer mediated learning: An example of an approach. *International journal of computers for mathematical learning*, *5*, 25–45.

Asiala, M., Cottrill, J., Dubinsky, E. & Schwingendorf, K. E. (1997). The development of students’ graphical understanding of the derivative. *The Journal of Mathematical Behavior*, *16*, 399–431.

Balchin, W. G. V. & Coleman, A. M. (1966). Graphicacy Should Be The Fourth Ace In The Pack. *The Cartographer*, *3*, 23–28.

Barton, R. (1998). Why do we ask pupils to plot graphs? *Physics Education*, *33*, 366–342.

Beichner, R. J. (1994). Testing Student Interpretation of Kinematic Graphs. *American Journal of Physics*, *62*, 750–762.

Bell, A. & Janvier, C. (Claude). (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, *2*, 34–42. Retrieved from http://www.jstor.org/stable/40240746

Bonanno, A., Bozzo, G., Camarca, M. & Sapia, P. (2011). Using a pc and external media to quantitatively investigate electromagnetic induction. *Physics Education*, *46*, 385.

Bowen, M. G. & Roth, W.-M. (1998). Lecturing graphing: What features of lectures contribute to student difficulties in learning to interpret graph? *Research in Science Education*, *28*.

Bowen, M. G. & Roth, W.-M. (2005). Data and graph interpretation practices among preservice science teachers. *Journal of Research in Science Teaching*, *42*, 1063–1088. doi:10.1002/tea.20086

Brasell, H. M. & Rowe, M. B. (1993). Graphing skills among high school physics students. *School Science and Mathematics*, *93*, 63–70.

Cleveland, W. S. (1984). Graphs in Scientific Publications. *American Statistician*, *38*, 261–69.

Cleveland, W. S. (1993). A model for studying display methods of statistical graphics. *Journal of Computational and Graphical Statistics*, *2*, 323–343.

Cleveland, W. S. & McGill, R. (1984). Graphical Perception: Theory, Experimentation, and Application to the Development of Graphical Methods. *Journal of the American Statistical Association*, *79*, 531–554.

Cleveland, W. S. & McGill, R. (1985). Graphical Perception and Graphical Methods for Analyzing Scientific Data. *Science*, *229*, 828–833.

Cook, D. R. & Weisberg, S. (1999). Graphs in Statistical Analysis: Is Medium The Message? *The American Statistician*, *53*, 29–37.

Cook, M. P. (2006). Visual representations in science education: The influence of prior knowledge and cognitive load theory on instructional design principles. *Science education*, *90*, 1073–1091.

Curcio, F. R. (1987). Comprehension of Mathematical Relationships Expressed in Graphs. *Journal of Research in Mathematics Education*, *18*, 382–393.

Deacon, C. (1999). The Importance of Graphs in undergraduate Physics. *The Physics Teacher*, *37*, 270–274.

Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2013). Measuring the mustard seed: A first exercise in mathematical modelling. In A. Jamakhandi, E. M. Sam & G. Nagarjuna (Eds.), *Proceedings of epiSTEME 5: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 213–219). CinnamonTeal Publishing.

Dhakulkar, A., Dhurde, S. & Nagarjuna, G. (2015). Measuring the mustard seed: an exercise in indirect measurement and mathematical modelling. *School Science Review*, *96*, 63–68.

Dhakulkar, A. & Nagarjuna, G. (2011). An Analysis of Graphs in School Textbooks. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 127–131). Macmillan.

Dhakulkar, A. & Nagarjuna, G. (G.). (2011a). Epicyclical Astronomy: A Case for GeoGebra. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 324–328). Macmillan.

Dhakulkar, A. & Nagarjuna, G. (G.). (2011b). From Geography to physics: How does geography help students learn motion? In S. Hellmann, P. Frischmuth, S. Auer & D. Dietrich (Eds.), *Proceedings of the 6th Open Knowledge Conference, OKCon 2011, Berlin, Germany, June 30 & July 1, 2011.* (Vol. 739). CEUR Workshop Proceedings. CEUR-WS.org.

Dhakulkar, A. & Nagarjuna, G. (2015). Exploring the phenomena of electromagnetic induction. In S. Chandrasekharan, S. Murthy, G. Banerjee & A. Muralidhar (Eds.), *Proceedings of epiSTEME 6: International Conference to Review Research on Science, Technology and Mathematics Education* (pp. 276–284). CinnamonTeal Publishing.

Dori, Y. J. & Sasson, I. (2008). Chemical understanding and graphing skills in an honors case-based computerized chemistry laboratory environment: The value of bidirectional visual and textual representations. *Journal of Research in Science Teaching*, *45*, 219–250.

Eshach, H. (2014). The use of intuitive rules in interpreting students' difficulties in reading and creating kinematic graphs. *Canadian Journal of Physics*, *92*, 1–8. doi:10.1139/cjp-2013-0369

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for research in mathematics education*, 94–116.

Even, R. (1998). Factors involved in linking representations of functions. *The Journal of Mathematical Behavior*, *17*, 105–121.

Friel, S. N., Curcio, F. R. & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in mathematics Education*, 124–158.

Gallagher, J. J. (1979). Basic skills common to science and mathematics. *School Science and Mathematics*, *79*, 555–565. doi:10.1111/j.1949-8594.1979.tb13894.x

González, M. T., Espinel, M. C. & Ainley, J. (2011). Teachers' graphical competence. In *Teaching statistics in school mathematics-challenges for teaching and teacher education* (pp. 187–197). Springer.

Hetland, L., Winner, E., Veenema, S., Sheridan, K. M. & Perkins, D. N. (2007). *Studio Thinking: The Real Benefits of Visual Arts Education.* Teachers College Press.

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *The Journal of Mathematical Behavior*, *17*, 123–134.

Hutchison, L., Ellsworth, J. & Yovich, S. (2000). Third-grade students investigate and represent data. *Early Childhood Education Journal*, *27*, 213–218.

Janvier, C. & Bednarz, N. (1989). Representation and contextualization. In *Proceedings of the 13th annual conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 139–146).

Kingman, R., Rowland, S. C. & Popescu, S. (2002). An experimental observation of faraday's law of induction. *American Journal of Physics*, *70*, 595–598.

Kosslyn, S. M. (1989). Understanding Charts and Graphs. *Applied Cognitive Psychology*, *3*, 185–225.

Kozhevnikov, M. & Thornton, R. (2006). Real-Time Data Display, Spatial Visualization Ability, and Learning Force and Motion Concepts. *Journal of Science Education and Technology*, *15*, pp. 111–132.

Krohn, R. (1991). Why are graphs so central in science? *Biology and philosophy*, *6*, 181–203.

Kumar, K. (1988). Origins of India's "Textbook Culture". *Comparative Education Review*, *32*, 452–464.

Latour, B. & Woolgar, S. (1986). *Laboratory Life: The Construction of Scientific Facts*. Princeton.

Lehrer, R. & Romberg, T. (1996). Exploring children's data modeling. *Cognition and Instruction*, *14*, 69–108.

Leinhardt, G., Zaslavsky, O. & Stein, M. K. (1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching. *Review of Educational Research*, *60*, 1–64.

McDermott, L. C., Rosenquist, M. L. & van Zee, E. H. (1987). Students difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics*, *55*, 503–513.

Mevarech, Z. R. & Kramasky, B. (1997). From Verbal Descriptions To Graphic Representations: Stability and Change in Students' Alternative Conceptions. *Educational Studies in Mathematics*, *32*, 229–263.

Mokros, J. R. & Tinker, R. F. (1987). The impact of microcomputer-based labs on children's ability to interpret graphs. *Journal of research in science teaching*, *24*, 369–383.

Monk, S. (2003). A Research Companion to Principles and Standards For School Mathematics. (Chap. 17 Representation in School Mathematics: Learning to Graph and Graphing to Learn, pp. 250–262). NCTM.

Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *The Journal of the Learning Sciences*, *5*, 239–277.

Moschkovich, J. N., Schoenfeld, A. & Arcavi, A. H. (1993). Chapter = Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them., In T. A. Romberg, E. Fennema & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (Chap. Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. pp. 69–100). Lawrence Erlbaum.

Nachmias, R. & Linn, M. C. (1987). Evaluations of science laboratory data: The role of computer-presented information. *Journal of research in science teaching*, *24*, 491–506.

NCERT. (2005). *National Curriculum Framework 2005*. National Council of Educational Research and Training.

NCTM. (2000). *Principles and standards for school mathematics* (3rd). National Council of Teachers of Mathematics.

Padilla, M. J., McKenzie, D. L. & Shaw, E. L. (1986). An examination of the line graphing ability of students in grades seven through twelve. *School Science and Mathematics*, *86*, 20–26. doi:10.1111/j.1949-8594.1986.tb11581.x

Paoletti, G. A. (2007). Problems in the integration of text and graphs. Retrieved from http://hdl.handle.net/10077/2545

Papert, S. (1980). *Mindstorms: Children, Computers, and Powerful Ideas*. New York: Basic Books.

Papert, S. & Harel, I. (1991). *Constructionism*. Ablex Publishing Corporation.

Pechenik Jan A, T. J. S. (1992). The Graphing Detective: An Exercise in Critical Reading, Experimental Design and Data Analysis. *The American Biology Teacher*, *54*, 432–434.

Peden, B. F. & Hausmann, S. E. (2000). Data Graphs in Introductory and Upper Level Psychology Textbooks: A Content Analysis. *Teaching of Psychology*, *27*, 93–97.

Pereira-Mendoza, L. (1995). Graphing in the primary school. *Teaching Statistics*, *17*, 2–6.

Pereira-Mendoza, L. & Mellor, J. (1990). Students' Concepts of Bar Graphs - Some Prelimnary Findings. In *ICOTS 3* (pp. 150–157).

Phillips, R. J. (1997). Can juniors read graphs? a review and analysis of some computer-based activities. *Journal of Information Techology for Teacher Education*, *6*, 49–58. doi:10.1080/14759399700200005

Pingle, F. (1999). *UNESCO Guidebook on Textbook Research and Textbook Revision.* Verlag Hahnsche Buchhandlung.

Pinker, S. (1990). Chapter 4 A Theory of Graph Comprehension. In R. Freedle (Ed.), *Artificial Intelligence and Future of Testing*. Lawrence Erlbaum.

Pratt, D. (1995). Young children's active and passive graphing. *Journal of Computer Assisted Learning*, *11*. doi:10.1111/j.1365-2729.1995.tb00130.x

Roth, W.-M. (1996). Where is the context in contextual word problem?: Mathematical practices and products in grade 8 students' answers to story problems. *Cognition and Instruction*, *14*. doi:10.1207/s1532690xci1404_3

Roth, W.-M. & Bowen, M. G. (2000). Learning difficulties related to graphing: A hermeneutic phenomenological perspective. *Research in Science Education*, *30*, 123–139.

Roth, W.-M. & Bowen, M. G. (2001). Professionals Read Graphs: A Semiotic Analysis. *Journal of Research in Mathematics Education*, *32*, 159–194.

Roth, W.-M. & Bowen, M. G. (2003). When are graphs worth ten thousand words? an expert-expert study. *Cognition and Instruction*, *21*, 429–473.

Roth, W.-M., Bowen, M. G. & McGinn, M. K. (1999). Interpretations of Graphs by University Biology Students and Practicing Scientists: Toward a Social Practice View of Scientific Representation Practices. *Journal of Research in Science Teaching*, *36*, 1020–1043.

Roth, W.-M. & Hwang, S. (2006). On the relation of abstract and concrete in scientists' graph interpretations: A case study. *Journal Of Mathematical Behavior*, *25*, 318–333.

Roth, W.-M. & McGinn, M. K. (Michelle K). (1997). Graphing: Cognitive ability or practice? *Science Education*, *81*, 91–106.

Roth, W.-M. & McGinn, M. K. (Michelle K.). (1998). Inscriptions: Toward a theory of representing as social practice. *Review of educational research*, *68*, 35–59.

Roth, W.-M., Pozzer-Ardenghi, L. & Han, J. Y. (2005). *Critical Graphicacy.* Science and Technology Education Library. Springer.

Shah, P. & Hoeffner, J. (2002). Review of Graph Comprehension Research: Implications for Instruction. *Educational Psychology Review*, *14*, 47–69.

Shah, P., Mayer, R. E. & Hegarty, M. (1999). Graphs as Aids to Knowledge Construction: Signaling Techniques for Guiding the Process of Graph Comprehension. *Journal of Educational Psychology*, *91*, 690–702.

Simkin, D. K. & Hastie, R. (1987). An information processing analysis of graph perception. *Journal of the American Statistical Association*, *82*, 454–465.

Smith, L. D., Best, L. A., Stubbs, D. A., Johnston, J. & Archibald, A. B. (2000). Scientific Graphs and Hierarchy of the Sciences : A Latourian Survey of Inscritption Practices. *Social Studies of Science*, *30*, 73–94.

Tairab, H. H., Khalaf, A.-N. & Ali, K. (2004). How do secondary school science students interpret and construct scientific graphs? *Journal of Biological Education*, *38*. doi:10.1080/00219266.2004.9655920

Tal, A. & Wansink, B. (2016). Blinded with science: Trivial graphs and formulas increase ad persuasiveness and belief in product efficacy. *Public Understanding of Science*, *25*, 117–125.

Tufte, E. (2001). *The Visual Display of Quantitative Information* (2nd ed.). Connecticut, USA: Graphics Press.

Wainer, H. (1984). How to display data badly. *The American Statistician*, *38*, 137–147.

Wainer, H. (1992). Understanding Graphs and Tables. *Educational Researcher*, *21*, 14–23.

Wainer, H. (2007). *Graphic Discovery: A Trout in the Milk and Other Visual Adventures*. Princeton University Press.

Wavering, M. J. (1989). Logical reasoning necessary to make line graphs. *Journal of Research in Science Teaching*, *26*, 373–379.

Wemyss, T. & van Kampen, P. (2013). Categorization of first-year university students' interpretations of numerical linear distance-time graphs. *Phys. Rev. ST Phys. Educ. Res. 9*, 010107. doi:10.1103/PhysRevSTPER.9.010107