Development and evaluation of a concept inventory in rotational kinematics

A Thesis

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by

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We have arranged a civilization in which most crucial elements profoundly depend on science and technology. We have also arranged things so that almost no one understands science and technology. This is a prescription for disaster...Sooner or later this combustible mixture of ignorance and power is going to blow up in our faces.

— Carl Sagan,
The Demon-Haunted World: Science as a Candle in the Dark, p. 26
(Random House, New York, 1995).

Dedicated to my grandparents
Kalarattu Kandiyil Moosa Haji and Khadeeja

Declaration

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions. The work was done under the guidance of Professor Vijay A. Singh, at the Tata Institute of Fundamental Research, Mumbai.

Mashood K. K.

In my capacity as supervisor of the candidate's thesis, I certify that the above statements are true to the best of my knowledge.

Prof. Vijay A. Singh

Date:

Acknowledgment

It was a long walk. Perhaps the longest which I ever undertook in my life consciously. Looking back to express my gratitude to all those who made this journey possible, I am at a loss on where to begin. Even the logical beginning from my parents seems inadequate. The work would not have been possible without my supervisor, Prof. Vijay Singh. I am grateful for his continuous and scholarly guidance.

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I thank all my teachers, friends and well wishers, from my childhood till date, for their support. I thank Charles Jose and Manoj Nair for their help. Let me conclude by admitting my conviction that a genuine gratitude should express itself in ways more meaningful than words. I pray that my life, knowledge and skills manifest accordingly.

Abstract

This thesis describes the research involved in the development and evaluation of a concept inventory (CI) in rotational kinematics at the higher secondary school level. A CI essentially comprises of a carefully crafted set of multiple choice questions (also called items) on a concept or a topic aimed at probing student difficulties, misconceptions or alternative conceptions, and eliciting their ill-suited reasoning patterns (Singh, 2011). They constitute a major trend in the field of physics education research (PER) (Kumar, 2011). Our concept inventory comprises of three parts as stated below and developed broadly in the same order.

1. Rotational kinematics of a particle (19 questions) (Mashood and Singh, 2012a).
2. Rotational kinematics of a particle in rectilinear motion (7 questions) (Mashood and Singh, 2012b, 2012c).
3. Rotational kinematics of a rigid body rotating about a fixed axis (13 questions) (Mashood and Singh, 2014).

The inventory was constructed systematically and iteratively. The processes involved in the initial phase of development can be categorized into (a) theoretical analysis, which constitute content mapping, cognitive analysis and literature review, (b) empirical investigations which include interactions with students and teachers. The theoretical analyses led to a preliminary draft of questions (open ended, true-false, explanatory type etc.) which evolved iteratively in the course of our interactions with students and teachers. In this initial phase we interacted with around 50 students and 12 practicing teachers. As part of the exercise we also taught rotational motion to a group of students. Verbal data was collected using think aloud protocol, retrospective probing and semi-structured interviews.

The items were then validated before administration to larger samples. Face validated was carried out by 10 practicing teachers, 10 higher secondary students and 8 graduate students in physics. The content validity was established by 8 faculty who taught physics at the university level. Before the first phase of administration the inventory was pilot tested on a group of 58 undergraduate students. They were asked to write down brief explanations for their answer choices.

The inventory was administered to four groups of students and two groups of teachers. A subset of the samples were interviewed. In the first stage of administration the student groups comprised of 79 (S1) and 74 (S2) candidates respectively. The entire inventory was further administered to a large sample of over nine hundred students (S3, N=905) from 5 urban centers (Jaipur, Patna, Mumbai, Hyderabad and Bangalore) spread across the country. All the students were at the higher secondary school level (HSS) and had a course on rotational motion. A part of the inventory was also administered to 384 (S4) introductory level students at the University of Washington, Seattle. The teachers taught physics at the HSS or undergraduate level. The two groups comprised of 26 (T1) and 25 (T2) candidates respectively.

Analysis of the response patterns and interviews revealed an array of misconceptions and pitfalls pertaining to rotational kinematics. These include student difficulty in comprehending the counter intuitive direction of angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$), their notion that $\vec{\omega}$ mimics the behavior of $\vec{\alpha}$, lack of knowledge of the validity conditions associated with equations like $\vec{v} = \vec{\omega} \times \vec{r}$, $\vec{a} = \vec{\alpha} \times \vec{r}$ and $\tau = I\alpha$ (here the symbols have their usual meaning), reluctance to ascribe angular quantities when the trajectory of motion is a straight line, the incorrect assumption that angular acceleration cannot exist without a net torque, among others. These misconceptions and pitfalls were further categorized into broader patterns of thinking prevalent among students which include fixation with inappropriate prototypes, indiscriminate use of equations, pitfalls paralleling those found earlier in linear kinematics, inappropriate extension of familiar procedural practices, reasoning cued by primitive elements in thought, and lack of differentiation between related but distinct concepts. Administration of part of the inventory to students at the University of Washington, Seattle indicated similarities between the response patterns of Indian and American students. Our study highlights the stark necessity for meaningful teacher professional development programs in the country.

The large scale administration made possible detailed statistical and item response curve (IRC) analyses of the inventory. Item-wise statistics namely the difficulty level, index of discrimination and point biserial coefficient were calculated. Ferguson's delta and Kuder Richardson reliability index were calculated to evaluate the discriminatory power and reliability of the inventory. IRCs were plotted for all 39 items and analyzed. The statistical and IRC analyses attested to the quality of individual items as well as the whole test. During the large scale administration we also administered two internationally standardized inventories namely the Force Concept Inventory and the Conceptual Survey on Electricity and Magnetism. The values of Ferguson's delta and Kuder Richardson reliability index calculated for these inventories were consistent with the values obtained for our inventory. Further, the results from our studies were communicated to international peer reviewed journals at appropriate stages of our study. The publication of the results ensured that the research was proceeding in the right direction.

A pedagogical spin-off, namely the variation of angular velocity and angular acceleration of a particle moving in a straight line with constant acceleration (origin not on the line of motion) was uncovered during the course of inventory development. The non-monotonic behavior of ω and α with associated extremum is interesting. We briefly discuss the pedagogical implications of our study. An operational definition for the angular velocity of a particle was suggested. The relevance of the present work in the Indian educational scenario and avenues for further research are explored.

List of Publications

PUBLICATIONS IN JOURNAL

1. Mashood, K.K. and Singh, Vijay A. (2012). An inventory on rotational kinematics of a particle: Misconceptions and pitfalls in reasoning. *European Journal of Physics*, *33*, 1301-1312.

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PUBLICATIONS NOT RELATED TO THESIS

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2. Mashood, K.K. and Singh, Vijay A. (2013). Large-scale studies on the transferability of general problem-solving skills and the pedagogic potential of physics. *Physics Education (IOP-UK)*, *48*, 629-35.

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Chapter 1

Concept Inventories: An Introduction

1.1 Introduction

Quality science education to a larger populace is integral to our economic and societal progress. Undoubtedly the challenges that we face in this regard are enormous. One should adopt a systematic approach towards identifying and addressing the underlying problems. This constitutes the broad goal of science education research (SER). The complex problems pertaining to teaching and learning of science are addressed by adhering to the procedures and rules of evidence characterizing scientific research as closely as possible (Wieman, 2007; McDermott, 2013). SER is a nascent area in India. In the west however, particularly in the United States, the discipline has gathered substantial momentum. The field owes its origin mainly to the theoretical insights on learning which emerged nearly half a century ago (Kumar, 2011). The pioneering work of Jean Piaget along with others like Lev Vygotsky are notable in this regard. Physics education research (PER) is a sub-discipline of this field. As evident from the term it focuses on aspects related to the teaching and learning of physics. PER has undergone tremendous growth in the past few decades. Areas of research include conceptual understanding, epistemology, problem solving, concept inventories, affective aspects, attitudes, social aspects and use of technology among others (McDermott and Redish, 1999; Beichner, 2009; Kumar,2011). Our study on rotational kinematics falls under the concept inventory category in PER.

A CI essentially comprises of a carefully crafted set of multiple choice questions (MCQs) on a concept or a topic aimed at probing student difficulties, misconceptions or alternative conceptions, and eliciting their ill-suited reasoning patterns (Singh, 2011). They constitute a major trend in the field of physics education research (PER) (Kumar,2011). Inventory questions/items should be distinguished from the MCQs that form a part of admission tests like the Indian Institute of Technology - Joint Entrance Examination (IIT-JEE). Inventories are not designed to grade individual students and act as a toll gate for placements. Rather they emerged in science education as continuation of the studies on student conceptions of topics in physics (Kumar, 2011). In addition to its utility as a diagnostic tool to identify student misconceptions and pitfalls, they serve to evaluate the effectiveness of instructional techniques and pedagogies. The potential for large scale application, rapid and easy evaluation, objective and quantitative data are among the merits of a CI. We will discuss aspects pertaining to inventories further in the coming sections. In what that follows we review literature on concept inventories and rotational motion. We also discuss the potential of CIs in the Indian context. We end the chapter by providing an overview of our inventory on rotational kinematics.

1.2 Concept inventories: Early history

The history of concept inventories in science education research can be traced back to the Force Concept Inventory (FCI) published in 1992 along with the Mechanics Baseline Test (MBT) (Hestenes, Wells, and Swackhammer, 1992; Hestenes and Wells, 1992; Richardson, 2004; Hake, 2011). The prequel to these inventories appeared earlier in 1985 (Halloun and Hestenes, 1985a, 1985b). FCI consists of 30 items probing students preconceptions which are incompatible with Newtonian mechanics. The subtopics include linear kinematics, Newton's three laws of motion, different types of forces and vector sum of these forces. MBT comprises of 26 items probing student understanding of the formal knowledge of these topics. These two inventories investigate two complementing dimensions of student understanding pertaining to basic mechanics. FCI items and distractors are meaningful even to those who have not learned any physics. In contrast formal knowledge of the topic is a prerequisite to comprehend and answer the MBT items (terms items and questions are used interchangeably).

The complementary probing of two dimensions of the same topic reflects a perspective concerning student cognition of physics concepts (Arons, 1990; Reif and Allen, 1992; Grayson et al., 2001; Reif, 2008). Broadly the sources of student difficulties were considered to be rooted in following two major factors.

1. The prior knowledge state of the learner: They are primarily characterized by naive notions concerning the physical world. These notions are often tacitly acquired over years of informal interactions with the physical world. Successful application of them in situations encountered in daily life makes them deep rooted. They do not yield very easily to formal instruction.

2. The form of the knowledge structure of physics: Physics and science in general, are designed deliberately to ensure maximum generality. The standards on precision, coherence etc are stringent. Novices stand in need of cognitive skills hitherto unlearnt to handle these complexities.

The inventories, particularly FCI, played a significant role in stimulating research driven educational reforms in physics (Richardson, 2004; Hake, 2011). Hake (2011) has provided a review of the impact inventories had on physics education and related disciplines. He noted that,

For more than three decades, physics education researchers have repeatedly shown that traditional introductory physics courses with passive student lectures, recipe labs, and algorithmic problem exams are of limited value in enhancing students' conceptual understanding of the subject (McDermott and Redish, 1999). Unfortunately, this work was largely ignored by the physics and education communities until Halloun and Hestenes (1985a,b) devised the Mechanics Diagnostic (MD) test of conceptual understanding of Newtonian mechanics. (Hake, 2002)

Owing particularly to the limited outreach of PER he characterized the pre-inventory period as the 'dark ages of post secondary physics education' in the United States.

FCI acted as an eye opener for many traditional instructors. The experience of the Harvard physicist Eric Mazur is an illustrative example. Administration of the inventory to his students gave him a surprisingly bleak picture of their understanding of basic concepts. The experience compelled him to incorporate research driven educational practices in his teaching. In the following excerpt Mazur describes his experience of trying the FCI with his students (Richardson, 2004).

I was entirely oblivious to this problem. I now wonder how I could be fooled into thinking I did a credible job teaching Introductory Physics. While several leading physicists have written on this problem, I believe most instructors are still unaware of it. A first step in remedying this situation is to expose the problem in one's own class. The key, I believe, is to ask simple questions that focus on single concepts. The result is guaranteed to be an eye- opener even for seasoned teachers.

Mazur (2007) later developed a pedagogy emphasizing interactive teaching in contrast to passive lecturing. The pedagogy known as 'Peer Instruction' is currently one of the most popularly employed research driven teaching method. The effectiveness and success of FCI further led to the development of inventories in physics as well as other areas of science and engineering. In the next section we review major inventories, with focus on physics.

1.3 Concept inventories: A literature survey

Major inventories in physics include Test of Understanding Graphs in Kinematics (TUG-K), Force and Motion Conceptual Evaluation (FMCE), Conceptual Survey of Electricity and Magnetism (CSEM), Brief Electricity and Magnetism Assessment (BEMA) and Student Understanding of Rotational and Rolling motion concepts, among others (Beichner, 1994; Thornton and Sokoloff, 1998; Maloney et al, 2001; Rimoldini and Singh, 2005; Ding et al, 2006). All these inventories were developed in the United States. Recently they are being developed in other parts of the world as well (Wuttiprom et al., 2009; Tongchai et al., 2009; Kohnle et al., 2011; Sharma and Ahluwalia, 2012; Aslanides and Savage, 2013). A useful list of CIs in physics has been provided by Biechner (2007). The topic-wise subcategories include graphing, force, mechanics, energy, thermodynamics, electricity and magnetism, light and optics, quantum mechanics, astronomy and waves. Table 1.1 provides a list of some of the inventories in physics.

Allen (2007) cataloged a similar list of inventories in the domains of engineering,

Table 1.1: List of some of the concept inventories in physics.

Concept Inventory	Authors (year)
Force Concept Inventory (FCI)	Hestenes, Wells and Swackhammer (1992)
Mechanics Baseline Test (MBT)	Hestenes and Wells (1992)
Test of Understanding Graphs in Kinematics (TUGK)	Beichner (1994)
Force and Motion Conceptual Evaluation	Thornton and Sokoloff (1998)
Conceptual Survey of Electricity and Magnetism (CSEM)	Maloney, O'Kuma, Hieggelke and Heuvelen (2001)
Multiple choice test of energy and momentum concepts	Singh and Rosengrant (2003)
Rotational and Rolling Motion Concepts	Rimoldini and Singh (2005)
Brief Electricity and Magnetism Assessment (BEMA)	Ding, Chabay, Sherwood and Beichner (2006)
Conceptual Survey in Introductory Quantum Physics	Wuttiprom, Sharma, Johnston, Chitaree and Soankwan (2009)
Diagnostic Test to Assess Secondary Students' Understanding of Waves	Caleon and Subramaniam (2010)
Conceptual Diagnostic Survey in Nuclear Physics	Kohnle, McLean and Aliotta (2011)
Alternative Conceptions of Fermi Energy	Sharma and Ahluwalia (2012)
Relativity Concept Inventory	Aslanides and Savage (2013)

chemistry, maths, geo-sciences etc. CIs have started playing an increasingly important role in biology education. Reviews of concept inventories in the subject are now available (D' Avanzo, 2008; Knight, 2010). Fisher and Williams (2011) have provided a list of CI's in various sub-disciplines of biology which include natural selection, genetics, introductory biology, molecular and cell biology. Some of the inventories in chemistry, biology, maths, astronomy and engineering are listed in table 1.2.

We briefly discuss the salient features of some of the prominent inventories in physics.

Test of Understanding Graphs in Kinematics, TUG-K (Beichner 1994):

TUG-K was developed by Beichner to probe student difficulties in interpreting graphs in linear kinematics. The inventory comprises of a set of 21 multiple choice questions. The sub-topics covered by the questions include position-time graph, velocity-time graph, acceleration-time graph and textual motion description. A number of difficulties were uncovered. Students tend to mistake graphs as pictures rather than as abstract representations. They believe that the graphs for distance, velocity and acceleration of motion under consideration will be identical to each other in appearance. Confusion between slope and height were also found. Value on the axis for height were thought to be the value of the slope. Students who correctly identified the slope of a line found the same difficult when the line was not passing through the origin. In terms of methodology, the paper presented a model for the construction of inventories. It provided a generic flowchart showing the various steps involved in the development of the inventory.

Force and Motion Conceptual Evaluation, FMCE (Thornton and Sokoloff, 1998)

FMCE comprising of 43 questions evaluates student understanding of basic Newtonian mechanics. More specifically the test covers one-dimensional kinematics and Newton's laws. The content covered is narrower than FCI. FCI includes two-dimensional motion with constant acceleration, vector addition, identification of forces etc, but FMCE does not. Both inventories also differ in terms of the representational format employed in the items. Verbal and graphical representation dominates FMCE. In contrast FCI relies more on verbal and pictorial representation (Thornton, et al., 2009). The construction of FMCE was accompanied by the development of a microcomputer based laboratory (MBL) curricula. The aim of this curricula was to help students develop a functional understanding of Newton's first and second laws.

Conceptual Survey of Electricity and Magnetism, CSEM (Maloney et al., 2001)

The inventory is a 32 item test to assess student knowledge of topics in electricity and magnetism. The content areas covered by CSEM include charge distribution on conductors/ insulators, Coulomb's law, electric field, principle of superposition, electric potential, work, induced charge, magnetic force, magnetic field by a current and Faraday's law. The authors noted that the number of studies on preconceptions in electricity and magnetism were limited, in contrast to mechanics. An array of pitfalls in understanding were uncovered. Poor understanding of charge distribution on conductors and insulators was noted. Students seemed to answer based on memorized statement about distribution of charge with little understanding of the physical mechanism. Another example is students' belief that a magnetic force acts on an electric charge whenever it is in a magnetic field. They often did not realize that the charge must have a velocity, with a component perpendicular to the direction of magnetic field. It was also found that many students failed to extend Newton's third law to electric and magnetic forces. The notion that the 'larger magnitude charge exerts larger force' on a smaller charge was popular.

Student understanding of rotational and rolling motion concepts (Rimoldini and Singh, 2005)

This inventory consists of 30 items investigating student difficulties with regard to concepts in rotational and rolling motion. The concepts covered include moment of inertia, angular velocity, angular acceleration, torque, rotational kinetic energy, motion on an inclined plane and rolling motion. It was found that students had difficulties with these concepts regardless of their level of mathematical knowledge. Some of the difficulties were related to the intricate nature of rotational motion. For example students considered force and torque to be equivalent as they were not clear about the concept of lever arm. They tend to think that two equal and opposite forces always imply a zero net torque. Some other difficulties could be traced back to related difficulties in linear motion. An illustration is the misconception in linear motion that 'constant net force implies a constant velocity'. Rotational analog of

Table 1.2: Some of the concept inventories in science and engineering disciplines.

Concept Inventory	Authors (year)
Chemical Concept Inventory (CCI)	Mulford and Robinson (2002)
Conceptual Inventory of Natural Selection (CINS)	Anderson, Fisher and Norman (2002)
A Concept Inventory for Molecular Life Sciences	Howitt, Anderson, Hamilton and Wright (2008)
Genetics Concept Assessment (GCA)	Smith, Wood and Knight (2008)
A Concept Inventory for Material and Energy Balances	Shallcross (2010)
The Astronomy and Space Science Concept Inventory	Sadler et al. (2010)
Geoscience Concept Inventory (GCI)	Libarkin, Anderson, Kortemeyer and Raeburn (2011)
Osmosis and Diffusion Conceptual Assessment	Fisher, Williams and Lineback (2011)
Chemical Engineering Fundamentals Concept Inventory (CEFCI)	Ngothai and Davis (2012)
Calculus Concept Inventory	Epstein (2013)

this namely that a ‘constant net torque implies a constant angular velocity’ was observed. The notion that friction always slows down motion is another example. We will discuss aspects of this inventory again in the next chapter while describing the development of our inventory on rotational kinematics.

1.4 Inventories: Focused vs broad survey instruments

An inventory can be broadly categorized as a focused or as a broad survey instrument, depending on the content it covers. It may be noted that the terms inventory and instrument are being used interchangeably. Broad survey instruments cover an entire topic like electricity and magnetism, nuclear physics etc. Examples include CSEM, the survey in nuclear physics and the inventory of rotational and rolling motion concepts (Maloney et al., 2001; Kohnle et al., 2011; Rimoldini and Singh, 2005). Some of them were discussed in the previous section. These instruments provide an overview of the student knowledge of the topic under consideration. Since most domains in physics build themselves upon an earlier topic (eg. special relativity on electromagnetism), the broad survey instruments have their merits. It gives instructors an idea of the knowledge state of the students in the concerned area. Since an entire topic like electricity and magnetism has to be covered, it is very likely that an item will involve two or more concepts. As a result what one obtains is an overview of student understanding of the topic. However we know that even elementary concepts like velocity and acceleration are rich in nuances (Trowbridge and McDermott, 1980, 1981; Reif and Allen, 1992; Shaffer and McDermott, 2005). Students experience an array of difficulties in comprehending these nuances pertaining to a single concept. This bring forth the need for focused inventories.

A focused inventory, in contrast to broad survey instruments, restricts itself to an in-depth investigation of a smaller domain or a smaller number of related concepts. Test for understanding graphs in kinematics (TUGK) discussed in the previous section is an example. It investigates student interpretation of graphs of position, velocity ($\vec{v}$) and acceleration ($\vec{a}$) with respect to time. Such in-depth investigations provide insights which can directly feed into teaching and inform curriculum development. They can also be adapted as clicker questions for interactive pedagogies like Peer Instruction.

Broad survey instruments as well as focused inventories can be used as pre-/post-tests. They serve as tools to evaluate teaching methods. As mentioned earlier CIs played a crucial role in establishing the effectiveness of interactive pedagogies over traditional modes of instruction. FCI and MBT data collected for over 6000 students showed significant gains in conceptual understanding and problem solving ability in interactive classrooms (Hake, 1998). Figure 1.1 illustrates how three types of classes, traditional and (moderate and strong) interactive, were evaluated using FCI (Redish, 2003). As the figure shows, the fractional gain $h =$ (class post-test average - class pre-test average) / (100 - class pre-test average), increased with interactive engagement in the classroom. It may be noted that interactive pedagogies are often developed by incorporating insights from science education research and cognitive science. Similar reforms and research driven educational practices are required in the India.

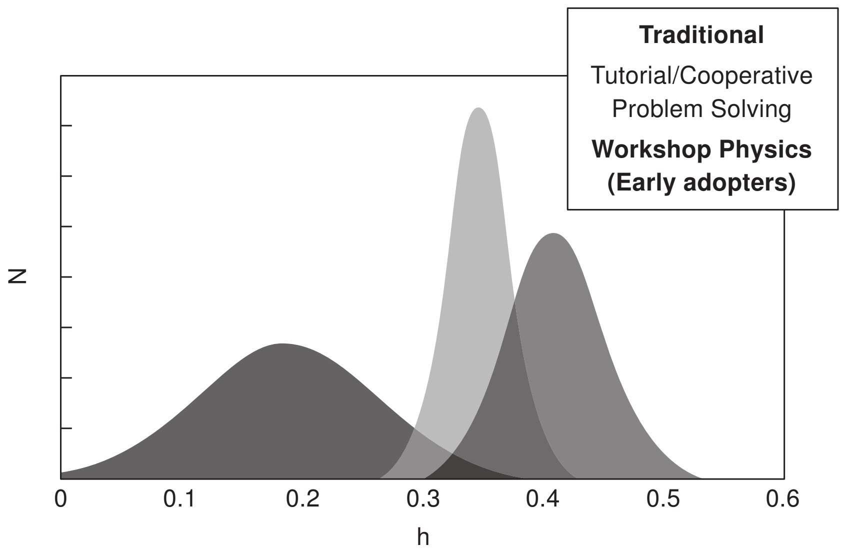

Figure 1.1: A plot of the fractional FCI gain (h) achieved in three types of classes: traditional (leftmost), moderate active engagement - tutorial/group problem solving (middle), and strong active engagement - early adopters of workshop physics (rightmost). Histograms are constructed for each group and fit with a Gaussian, which is then normalized [Source - figure and caption: Redish (2003)].

1.5 Indian scenario

India has a huge and diverse student and teacher population. But science education research has received scant attention from the academic community. The teaching and learning of science are seldom research driven. As mentioned earlier, we should adopt a scientific approach towards identifying and addressing the underlying problems. By a scientific approach we mean isolating specific problems, studying them using appropriate methodology or instruments, developing claims and arguments based on data rather than anecdote or untested personal convictions. The present work is an initiative towards achieving these broader goals. We focus on the higher secondary school (HSS) level. It may be noted that HSS plays a decisive role in the career trajectory of science students in India.

We decided to craft our research in the mode of a concept inventory because CI's can uniquely blend research and dissemination. Its potential for large scale application is particularly relevant to the Indian educational scene owing to our huge student population. A well developed CI, whose validity and reliability has been established, serves as a ready to use diagnostic and assessment tool. They can be administered to a large number of students at a time and evaluated easily and objectively. Student understanding and effectiveness of pedagogies /teaching can be evaluated. Our experiences reveal that classrooms across the country are still largely based on traditional lectures. Students are often passive recipients in this instructional format. We encountered numerous instances where the gap between what is taught and what is learnt is attributed to 'poor' students or teachers or both. Very few people are aware of even the basic findings of science education research such as the existence of alternative conceptions. Acquaintance with concept inventories can be an opportunity to educate students and teachers in this regard. It may be noted that distractors to well constructed CI items incorporate students' alternative conceptions and pitfalls. We recall many instances after administration of inventories like FCI where students and teachers remarked that they seldom encountered such questions. Similarly, exposure to other inventories in a variety of topics (see tables 1.1 and 1.2) can be an initial step towards a culture of learning where alternative conceptions are viewed as resources to achieve a better understanding. Our experience suggests that they are often looked down upon or dismissed without further consideration.

The linguistic, cultural and socio-economic diversity of India would make it a fertile ground for investigating alternative conceptions. As mentioned earlier, crafting them in the form of inventories would facilitate dissemination. The huge teacher population of the country is an untapped resource in this regard. Researchers in science education can collaborate with motivated teachers and develop a pool of inventories to make significant impact. Content experts from our universities can also form a part of the endeavor. Such an effort may provide momentum to science education research in the country. These initial steps can then be supplemented by transforming traditional classrooms to an interactive mode. Again the inventory items can be a facilitator as exemplified by pedagogies like peer instruction (Mazur, 2007). Items with good distractors that capture students' alternative notions can be resourceful in initiating peer interactions. The simplicity of multiple choice questions would help even less assertive students to participate in the process. Taking into account these possibilities we embarked upon the process of inventory development. We focused on rotational motion, the reason for which will be discussed in the next section.

1.6 Rotational motion in PER

We chose rotational motion because the topic has not yet received the attention it deserves from the physics education research community (Rimoldini and Singh, 2005). This is despite the fact that it is one of the most difficult topics at the higher secondary level, as revealed by our interactions with both students and teachers. The work by Rimoldini and Singh (2005), discussed above, was the first major effort to address this lacuna. There also exist scattered work on student understanding of the dynamics of rigid body rotation and rolling motion. We briefly review them in this section.

Physics education group at the University of Washington investigated aspects of dynamics of rotational motion. Part of the work was carried out while studying student understanding of static equilibrium (Ortiz et al., 2005). They found that students were relatively more comfortable analyzing point particles compared to a mass distribution. Aspects pertaining to the concepts of torque and center of mass were probed in the context of objects in static equilibrium. Students tend to believe that tilted orientation of bodies at rest is caused by unbalanced torques. Another naive notion concerned the center of mass (CoM). Students thought that CoM 'divides an object into two pieces of equal mass'. They also probed student understanding of the definition of torque involving a cross product. It was found that students experienced difficulties in differentiating force from torque. In general students difficulties were found to increase with slightly complicated mass distributions, in contrast to point particles and symmetrical shapes.

Another interesting work focused on the angular momentum of particles (Close and Heron, 2011). It was found that introductory students tend to ascribe zero angular momentum to a particle moving in straight line. The dependence of angular momentum of a particle on the choice of origin was not appreciated. In addition many students thought that angular momentum and linear momentum are related in such a way that increase in the former was at the expense of the latter. This was revealed by a context in which a ball hits a rod and sticks to it. Students were asked about the angular momentum of the ball-rod system in the following two cases; (a) ball hits the center of mass of the rod, (b) ball hits one of the ends of the rod. In general students attributed a lower linear momentum to the ball-rod system in case b compared to case a. In case b since the rod rotated after collision students thought that a part of the linear momentum contributed to the increase in angular momentum. Another related study investigated students' application of Newton's second law in the context of rigid body rotation (Close et al., 2013). The same force was applied on a rigid body at various points. In cases when the body rotated students erroneously thought that the force had a diminished effect on linear acceleration of the body. Force was being conceived as a quantity that gets 'used up' partly or fully as the body rotated.

There exist scattered studies probing student difficulties with rolling motion. Lopez (2003) investigated misconceptions pertaining to the relation between the linear and angular acceleration of a body rolling without slipping. Another work studied the effect of frictional force on rolling motion (Carvalho and Sousa, 2005). Our group at an earlier stage developed a concept inventory probing the direction of friction on rolling bodies (Singh and Pathak, 2007). Students and teachers experienced an array of difficulties understanding various aspects of rolling motion. In particular the fact that frictional force can sometimes be along the direction of motion were probed. However our further interactions with students and teachers revealed that they experienced difficulties in understanding even elemental concepts of rotational motion like angular velocity and angular acceleration. A review of literature made clear that studies on the concepts of angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$) were missing. The broad spectrum inventory by Rimoldini and Singh (2005) touches on a few aspects (which will be discussed in next the chapter). A minor study by Unsal (2011) involved the development of a low cost apparatus to help students understand angular speed. On the other hand linear velocity (v) and acceleration (a) have been the subject of repeated investigations (Trowbridge and McDermott, 1980, 1981; Halloun and Hestenes, 1985a, 1985b; Reif and Allen, 1992; Hestenes and Wells, 1992; Thornton and Sokoloff, 1998; Shaffer and McDermott, 2005). Our observation of the existence of difficulties among students as well as teachers regarding ($\vec{\omega}$) and ($\vec{\alpha}$), led us to research them in a focused manner.

1.7 A concept inventory on rotational kinematics

We have developed a concept inventory on rotational kinematics at the higher secondary school level. The following questions served as the basis of our research.

1. What difficulties do students face in rotational kinematics at the higher secondary school level?
2. Do the pitfalls in student reasoning in the topic exhibit patterns? Can they be categorized into broader themes within physics education research?
3. Are there any parallels between our findings in rotational kinematics and the documented research in linear kinematics?

We developed an inventory comprising of three parts and developed broadly in the order stated below.

1. Rotational kinematics of a particle (19 questions) (Mashood and Singh, 2012a).
2. Rotational kinematics of a particle in rectilinear motion (7 questions) (Mashood and Singh, 2012b, 2012c).
3. Rotational kinematics of a rigid body rotating about a fixed axis (13 questions) (Mashood and Singh, 2014).

The next chapter describes the methodological details involved in the systematic and iterative construction and administration of our inventory (Mashood and Singh, 2013a). In chapter 3 we discuss the first part of our inventory comprising 19 questions on rotational kinematics of a particle (Mashood and Singh, 2012a). The content evolution of items along with our findings is presented. Misconceptions and pitfalls were identified and categorized into broader themes within PER. Chapter 4 similarly discusses the next part of the inventory, namely rotational kinematics of a particle in rectilinear motion (Mashood and Singh, 2012b). This consists of 7 questions. Chapter 5 describes an interesting pedagogical spin-off related to the case of a particle moving in rectilinear motion (Mashood and Singh, 2012c). The non-intuitive variation of the angular velocity and the angular acceleration with associated extremum is the theme of the chapter. The final part of the inventory focusing on rotational kinematics of a rigid body about a fixed axis (13 questions) is discussed in chapter 6. We administered the entire inventory to around a thousand students in five cities spread across the country. The data from this large scale administration served as the basis for item level and whole test statistical analyses. This includes item response curve analyses and calculation of the Kuder-Richardson reliability index and other indices for the inventory. These statistical analyses constitute the subject of chapter 7. Chapter 8 constitutes the conclusion.

Chapter 2

Methodology

2.1 Introduction

The processes involved in the construction of our inventory on rotational kinematics can be arranged into three broad stages, namely an initial developmental phase followed by administration and evaluation. A schematic representation of these stages is given in figure 2.1 (Singh, 2011). The whole process was iterative as indicated by the loop in the figure. Only after the three stages have been completed and the inventory's validity and reliability is established, can it be disseminated. We note however that the items and distractors are open to refinement even after dissemination, in the face of new insights. Let us discuss the methodological details and intricacies involved in each of the three stages mentioned above.

2.2 Initial phase of development

In this section we discuss the steps involved in the initial phase of development of our inventory. Figure 2.2 schematically shows these various steps (Mashood and Singh, 2013). The processes involved can be categorized into (a) Theoretical analysis and (b) Empirical investigations. The theoretical analyses comprising of content mapping, cognitive analysis and literature review led to a preliminary draft of questions. Our experience and informal interactions with students and teachers also helped us in this regard. The preliminary draft of questions served as the basis for

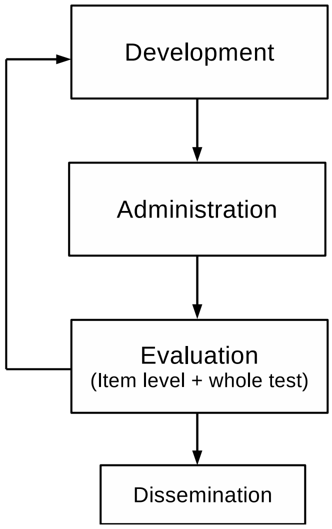

Figure 2.1: Schematic of stages involved in the construction of a concept inventory. Note the loop which signifies the iterative nature of the process.

our systematic interactions with students and teachers and these constitute empirical investigations. The aim was to obtain insights so that appropriate items and distractors could be framed and inappropriate ones discarded. We discuss each of the steps outlined in figure 2.2 in detail with illustrative examples.

2.2.1 Conception to content mapping

As discussed in chapter 1, we decided to develop a focused inventory on angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$ at the higher secondary school (HSS) level. The conception of the topic was based on the fact that it is an inadequately researched topic in physics education research (PER) and students harbor many misconceptions about it. The development began by chalking out aspects of $\vec{\omega}$ and $\vec{\alpha}$ covered at the HSS level. This process, which we refer to as content mapping, was done by analyzing presentations of rotational motion by various text books (Reif, 95; Halliday et al., 2001; Young and Freedman, 2004; Giancoli, 2005; NCERT, 2006). In addition two books in the vernacular language (Hindi) were consulted (Singh, 1988; Kumar and Mittal, 1991).

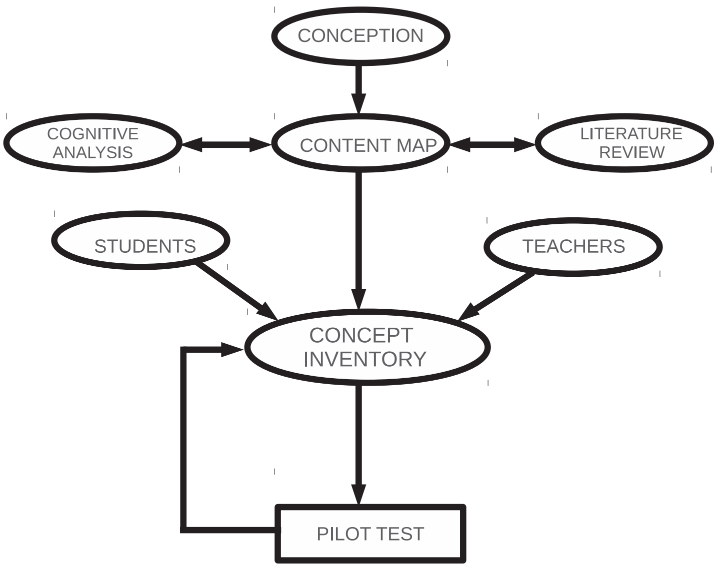

Figure 2.2: Processes involved in the initial phase of construction of the concept inventory. These steps are followed by validation, after which the inventory is administered to larger samples.

We found that the discussion of $\vec{\omega}$ and $\vec{\alpha}$ takes place mainly in the context of a rigid body. For a particle, it is often restricted to a brief discussion alongside uniform circular motion. This short shrift given to rotational kinematics of a particle may not be pedagogically prudent. A detailed treatment of the case of a particle is important and the broad structure of our inventory comprised of

- Rotational kinematics of a particle.
- Rotational kinematics of a rigid body about a fixed axis.

It was followed by identification aspects of $\vec{\omega}$ and $\vec{\alpha}$ that would be probed by the inventory. These aspects included

- Magnitude of angular velocity.
- Direction of angular velocity.
- Magnitude of angular acceleration.

Table 2.1: Aspects of $\vec{\omega}$ and $\vec{\alpha}$ probed by the current inventory and the items/questions probing them. The alphabet adjoining the question number in the right column indicates the correct answer. The items are listed in appendix A.

Magnitude of angular velocity of a particle	1c, 7a, 8c
Direction of angular velocity of a particle	2d, 5d, 8c, 20c
Discriminating angular velocity from angular position	3a
Concept of limit (single instant, single position)	3a, 7a, 14d
Change in angular velocity as the particle moves	4a, 9c, 10c, 11a, 19b
Validity of the equation $\vec{v} = \vec{\omega} \times \vec{r}$	6c, 19b
Magnitude of angular acceleration of a particle	12c, 14d, 15b, 21c, 22d
Direction of angular acceleration of a particle	12c, 15b, 21c, 24c
Change in angular acceleration as the particle moves	13b, 16d, 17d, 18d
Relation between angular acceleration and centripetal acceleration	36b, 37b
Relation between angular acceleration and tangential acceleration	35a, 37b
Validity of the equation $\vec{\tau} = I\vec{\alpha}$	23a, 33d
Components of linear velocity, linear acceleration	25a, 38c
Angular and linear velocities of particles on a rigid body	29b, 30d
Angular velocity of a rigid body	26c, 27c, 31a
Trajectory of an arbitrary particle on a rotating rigid body	28a
Angular acceleration of a rigid body	32b, 33d
Validity of the equation $\vec{a} = \vec{\alpha} \times \vec{r}$	36b, 39d
Origin dependence of angular velocity	3d, 34a

- Direction of angular acceleration.

- Relationship between linear and rotational kinematic variables.

Open ended and free response questions were designed around these conceptual dimensions. In the process of development these dimensions further branched out into specifics, as can be seen from table 2.1. The table provides a description of various aspects of $\vec{\omega}$ and $\vec{\alpha}$ covered by the inventory , along with the relevant questions. It may be noted that table is based on the latest version of inventory which had been developed systematically as discussed in the current chapter.

2.2.2 Cognitive analysis

The development of open ended and free response questions was guided by a cognitive analysis of $\vec{\omega}$ and $\vec{\alpha}$ akin to that done by Reif and Allen (1992) for linear acceleration. This process was facilitated by reviewing an undergraduate and an advanced level presentation of rotational motion (Kleppner and Kolenkow, 2007; Goldstein et al., 2004). It involved a 'theoretical analysis of the kinds of knowledge required to interpret the concepts effectively and efficiently' (Reif and Allen, 1992). Such an analysis unveils subtleties and nuances which are of pedagogical relevance. For linear acceleration it was pointed out that application of its definition would involve a sequence of steps even in the simplest cases. Each of these steps, though obvious to an expert, can prove to be a potential conceptual hurdle for a novice. The case of $\vec{\omega}$ and $\vec{\alpha}$ is similar. Consider the operational definition of angular velocity of a rigid body as an illustrative example. Identifying the angle $\Delta\theta$ in $\omega = \Delta\theta/\Delta t$ would require selecting an arbitrary particle on the body, not necessarily the center of mass, drawing a perpendicular line from the particle to the axis of rotation, noting the angle traced by this line as the rigid body rotates etc. As another example, consider the case of $\vec{\alpha}$ which may be non-zero even if instantaneous angular velocity is zero. Operationally this would entail, among other things, identifying angular velocities at two different instances and vectorially subtracting them. We noted other intricacies which helped us probe pitfalls in student thinking.

Other pedagogically significant aspects concerning $\vec{\omega}$ and $\vec{\alpha}$ which we noted include

1. The counter intuitive direction of the vectors $\vec{\omega}$ and $\vec{\alpha}$.
2. The derivations of commonly employed equations involving $\vec{\omega}$ and $\vec{\alpha}$ and their range of validity.
3. Origin dependence of the angular velocity of a particle.
4. Irrelevance of an origin in the definition of $\vec{\omega}$ for a rigid body, where the significant aspect is the axis of rotation.
5. Rigid body rotation about a fixed axis dealt at the introductory level is a special case of the general case described by Euler's theorem.

Open ended and free response questions (see appendix B for examples) were crafted incorporating these aspects. After interactions with students these questions evolved into multiple choice questions. Table 2.1 gives the question numbers along with the conceptual aspect being probed. The developed items are listed in appendix A.

2.2.3 Literature review

PER literature relevant to the topic was reviewed with the aim of obtaining insights that could feed into the development of questions. The literature review focused on works pertaining to

- Linear kinematics
- Rotational motion
- Prominent inventories in physics

We have reviewed works on rotational motion and inventories in physics in chapter 1. Here we will focus on studies which influenced the development of our items and distractors. As mentioned in chapter 1 linear kinematics has been repeatedly investigated (Trowbridge and McDermott, 1980, 1981; Halloun and Hestenes, 1985a, 1985b; Reif and Allen, 1992; Hestenes and Wells, 1992; Thornton and Sokoloff, 1998; Shaffer and McDermott, 2005). The study by Trowbridge and McDermott (1980) on linear velocity and linear acceleration uncovered interesting patterns in student thinking. Prominent among them is the non-discrimination between position and velocity where students think that bodies moving in parallel have the same velocity when they reach the same position. Students tend to focus on the 'perceptually obvious phenomenon of passing' rather than the procedure for identifying instantaneous speed. The general lack of differentiation between related concepts like position, velocity and acceleration was pointed out by many studies that followed.

Reif and Allen (1992) investigated student reasoning about velocity and acceleration. An oscillating simple pendulum was used as one of the contexts for posing questions. We realized that a pendulum can be a pedagogically rich context not only for probing linear kinematics, but also for rotational kinematics. A popular question was to probe both linear velocity and linear acceleration at the same point. Students were asked about the linear acceleration of the pendulum bob at the extreme point where its velocity is zero. The same concepts were probed in other contexts like a ball which is thrown up vertically. This motivated us to craft similar questions involving angular velocity and angular acceleration, which eventually resulted in items 7 and 14 (see appendix A).

We have discussed the broad survey instrument by Rimoldini and Singh (2005) in chapter 1. It may be noted that the terms instrument and inventory are used interchangeably. The instrument has a few questions on angular velocity and angular acceleration. However since it spans the whole area of rotational and rolling motion the questions in general involve two or three major concepts. For example consider question 5 which probes the effect of a non-zero net torque acting on a rigid body. The distractors involve angular velocity, angular acceleration and torque. In contrast question 13 was restricted to probing the variation of angular velocity of an oscillating rod of negligible mass with a mass M attached at one end. However the distractors to the question involved interpretation of graphs as well. This motivated us to probe student understanding of the variation of angular velocity. Since the variation could be in magnitude or direction, both the aspects were probed. Table 2.1 provides the number of questions on this topic in the inventory.

As discussed earlier textbooks were reviewed. This cued us to physically relevant situations like pendulum, elliptical or planetary motion etc wherein the questions could be posed. Analysis of the NCERT text book and vernacular textbooks along with our own experiences helped in figuring out contexts familiar to the Indian scenario such as wall clock, potters wheel, giant wheel etc to pose the questions. Most of our items were consciously framed in such physically relevant and familiar contexts. Questions from national level tests, namely the Indian Institute of Technology - Joint Entrance Examinations (IIT JEE), spanning twenty years were reviewed. An illustrative example would be the following problem on angular momentum which appeared in the 1997 IIT JEE (Pandey, 2011) .

1. A mass m is moving with a constant velocity along a line parallel to x-axis, away from the origin. Its angular momentum with respect to the origin

   (a) is zero. (b) remains constant. (c) goes on increasing. (d) goes on decreasing.

A similar problem was also part of a PER work (Close and Heron, 2011). This problem led us to investigate student understanding of the simpler concept of angular velocity of a particle in rectilinear motion.

The initial draft of questions did not have four choices for all items. Some were true - false types and some others open ended. Examples of them are listed in appendix B. It was the interactions with students and teachers that gave the items their final form. Methodological intricacies involved in those interactions along with illustrations of how some of the items evolved will be the theme of the next section.

2.3 Prior interactions and pilot studies: Methodological intricacies

Interaction with students and teachers is one of the important aspects involved in the process of developing an inventory. A knowledge of the thought processes of novice students helps significantly in constructing good questions/items (term questions and items are used interchangeably throughout) and distractors. We interacted with around 50 students and 12 practicing teachers before administering the test to a larger sample. The students comprised of 21 from the higher secondary level, 14 from bachelors level, 6 from masters level and 9 at the doctoral level. Some of the interactions were with small groups (2 - 6 students) while others were individual (Mashood and Singh, 2013a). The processes were iterative and began with the preliminary set of items/questions mentioned in the previous section. Verbal data was collected, primarily through the following modes.

1. Think aloud protocol: This involved candidates answering the questions and verbalizing their thinking at the same time. This was often followed by clarifications which progressed into discussions.

2. Retrospective probing: This involved students solving the questions and being probed by us at the end of the task (Young, 2005). Some students were more comfortable with this mode rather than the think aloud protocol in which they have to simultaneously solve and verbalize.

3. Semi-structured interviews: This involved candidates taking the test, marking their confidence level to each answer and then being interviewed.

All three of these were employed in the initial phase of development whereas only semi-structured interviews were employed later. The think aloud data, interviews and discussions were audio recorded and analyzed. The insights obtained were successively incorporated at each stage thereby refining the questions and distractors. Let us delve into some of the intricacies involved in the above mentioned methodologies. Think aloud protocol essentially comprises of the subject articulating their thoughts while solving a given problem. The method is particularly useful for providing insights during the early phases of investigation (Young, 2005). This makes it an apt tool in the developmental phase of a test. Cognitively it aims at capturing what is held in the short term memory (Ericsson and Simon, 1993). The primary aim is to elicit the sequence of thoughts as the subject is processing the information. As such, the researcher should restrict oneself to minimal intervention so that the stream of thought is not cued or influenced. We, like others, limited ourselves to minimal prompts or prodding such as 'keep talking', whenever the subject turned quiet (Young, 2005; Rimoldini and Singh, 2005). It is also important that the problems should be of optimal cognitive load (Young, 2005). A highly demanding problem makes it difficult for the participant to simultaneously attend to solving it and verbalizing. An extremely easy task may be performed with such automaticity that the subject may not be able to describe any sequence of steps. We tried to make our questions optimal in terms of difficulty and ensured that they could be answered without resorting to any lengthy algebraic manipulations. These issues are significant for retrospective probing as well. The individual differences in the ability to verbalize was taken into account. Students who found it difficult to solve and verbalize simultaneously opted for retrospective probing.

The cognitive analysis of the concepts discussed in the previous section helped structure our interviews. We tabulated a list of probable methods and arguments participants may invoke. As mentioned earlier the initial draft of open ended questions (see appendix B) evolved into multiple choice questions based on the interactions with students. These MCQs underwent pilot tests before they were shaped into the final form given in appendix A. In the pilot tests students were asked to assign confidence level to their choices, which further helped in interviews. A wrong answer with a high confidence level was indicative of the existence of misconceptions (Hasan et al, 1999). Thus the confidence level in conjunction with the answer choice hinted at potential focal points for interviews. The interviews were audio recorded and analyzed. The duration of each session ranged from 40 - 50 minutes.

In total there were around 30 open ended and free response questions in the initial draft. Examples of these are listed in appendix B. Interaction with students and teachers resulted in the development of around 80 multiple choice questions. There were multiple versions of questions probing the same student notions. Questions were dropped or modified during the course of development, winnowing down to the 39 items listed in appendix A. Faculties who conducted research in PER or taught physics at the university level were consulted in this regard. Illustrative examples of modifications made to items during the iterative process are given in appendix C. Appendix C also contains examples of items that were dropped along with reasons for the same. In the next section we discuss the details of the samples and some of the observations made during our interactions with students and teachers. Following that we give illustrative examples of the development of some of our items.

2.4 Prior interactions and pilot studies: Sample descriptions and observations

2.4.1 Higher secondary school level students

Interaction with teaching: 4 students, who had completed a course on particle mechanics were taught rotational mechanics by us. This was intended to obtain insights about the difficulties exhibited by novices while trying to comprehend $\vec{\omega}$ and $\vec{\alpha}$. While I taught, the supervisor carefully observed the classroom behavior such as questions, clarifications and responses to them. After teaching, the students were asked to answer questions by thinking aloud. Teaching scrupulously followed a text book, in order to avoid teaching to our items (NCERT, 2006).

Our observations, borne out of classroom interaction as well as analysis of audio recordings included

1. Difficulty in accepting that the direction of $\vec{\omega}$ was perpendicular to the plane of motion.

2. Ignoring the directional aspect of vectors.

3. Considering circular motion as the sole prototype of rotational motion.

4. Clumsy reasoning and retrieval, indicating a poor knowledge organization.

Interaction without teaching: This involved three students who had recently learnt rotational motion. Here two opted for retrospective probing. The observations were similar to those mentioned for the above group.

Interaction with olympiad aspirants: We interacted with 3 olympiad students, who answered the items by thinking aloud. All of them were physics or chemistry olympiad aspirants, selected for the final round. They answered most of the questions easily. Two were not very comfortable in verbalizing their thoughts and mostly provided answers without qualifiers. Notable errors were made regarding the validity of the equations $\vec{v} = \vec{\omega}\times\vec{r}$ and $\vec{\tau} = I\vec{\alpha}$. Here $\vec{v}$, $\vec{r}$, $\vec{\tau}$, I denote linear velocity, position vector, torque and moment of inertia respectively.

Pilot test 1: The questions were pilot tested with 7 students. All were asked to attach a confidence level to each answer on a 3 point scale. The performance suggested pitfalls in reasoning. Interviews affirmed the existence of robust erroneous notions like $\vec{\omega}$ lies in the plane of motion, rotational motion concepts are relevant only when the trajectory is circular or curvilinear etc.

Pilot test 2: Here 4 olympiad students answered the items. The confidence level was high for all items including a few questions which were answered incorrectly. Instances of high confidence level along with wrong answers indicated the existence of pitfalls, which were probed in detail during interviews. For example 3 students confidently maintained that the equation $\vec{v} = \vec{\omega}\times\vec{r}$ holds for all situations.

2.4.2 Undergraduate and post graduate students

Interaction: This comprised 6 undergraduate students and 2 post graduate students. Three preferred retrospective probing. The students were from regional colleges as well as national institutes like Indian Institute of Science Education Research

(IISER), Indian Institute of Technology (IIT) etc. The performance of students from regional colleges was unsatisfactory, at times displaying stark unfamiliarity with elementary concepts like $\vec{\omega}$ and $\vec{\alpha}$. Students from national institutes performed satisfactorily, some of them exceptionally well. Notable observations include a student answering questions in a variety of ways exhibiting strongly connected network of concepts and some others invoking unnecessary mathematics where simpler reasonings would have sufficed.

Pilot test 3: 8 undergraduate students and 4 post graduate underwent pilot tests. The interviews with the regional college students indicated difficulties which involved poor understanding of the concept of limit and confused reasonings for $\vec{\omega}$ and $\vec{\alpha}$ particularly in the context of a simple pendulum.

2.4.3 Doctoral students

Interaction: We interacted with 4 doctoral students in physics. Some of them were out of touch with topics. Nevertheless they were ready to reason out and this at times led to digression to advanced aspects of the topic. Discussion of Euler's theorem , Chasle's theorem etc provided us with wider perspectives. Inspired by this, we crafted a few items on the basic definition of $\vec{\omega}$ for a rigid body.

Pilot test 4: Involved 5 doctoral students. Some similarity was observed with the errors of olympiad aspirants.

2.4.4 Practicing teachers

We interacted with about 12 teachers. They shared their classroom experiences and aspects of the topics which they found difficult. This included the vector nature of the rotational concepts, mathematical operations like cross products, transition from single particle to rigid body etc. The discussion mostly proceeded informally unlike with students. Some experienced teachers suggested problems and questions which inspired items for our inventory.

2.4.5 Pilot test 5

The multiple choice questions before administration (to the samples discussed below in section 7) was pilot tested on a group of 58 undergraduate students. They were asked to write brief written explanations to their answer choices. The coherence between the explanations and answer choices were examined. Insights from these explanations fed into refinement of items and distractors.

2.5 Development of questions: Illustrative examples

In this section we discuss how the student interviews helped open ended questions evolve into the multiple choice format given in appendix A. In what follows 'I' stands for the interviewer and 'S' for the student.

2.5.1 Development of item 2 - Appendix A: Interview excerpts

A familiar artifact to the Indian students is the wall clock. We decided to probe aspects of angular velocity using it as a context. Item 2 probes student knowledge regarding the direction of angular velocity. It asks about the direction of the angular velocity of the second hand of a transparent clock when viewed from both the front and the rear. Below we provide excerpts from a student interview that helped craft the item and its distractors.

I: What can you say about the motion of the clock hands ?

S1: The second hand moves faster than the other two hands.

I: Let us ignore the hour hand and focus only on the minute hand and second hand. Assume that they are moving continuously. Which one has a greater angular velocity?

S1: Second hand has a greater angular velocity since it is moving faster.

I: What about the direction of angular velocity?

S1: (Student draws a clock on a paper. Talking to himself.....Murmurs....'clockwise'.... Simultaneously making gestures using hand.) Direction of angular velocity will be into the paper (shows the direction pointing fingers into the paper).

I: Why?

S1: Using right hand rule.....

I: Can you illustrate.

S1: The second hand is always moving clockwise. (Starts using right hand to show the orientation of motion). That means the thumb will be pointing into the paper. So direction of angular velocity will be into the paper.

I: Suppose it is a transparent clock. How will the second hand appear if you look at it from the back side.

S1: (Thinks for a moment. Uses gestures with right hand. Finger drawing arcs clockwise and anticlockwise). From the backside the second hand will be moving anticlockwise.

I: What about the direction of angular velocity, if you are looking from the backside?

S1: It will be opposite......When the second hand moves clockwise and the direction of angular velocity is into the plane of paper....from the back it will be moving anticlockwise...so the direction will be opposite......It will be out of the plane of paper (tries to illustrate the flipping of direction using right hand).

2.5.2 Development of items 19, 23 - Appendix A: Interview excerpts

Particle moving in a straight line is a pedagogically rich area to probe rotational kinematics. As mentioned earlier, studies have probed student understanding of angular momentum in the same context (Close and Heron, 2011). We developed a series of questions to investigate student understanding of angular velocity and angular acceleration in the context. Given below are excerpts from a student interview that helped develop items 19 and 23.

I: A particle is moving in a straight line. Can it have an angular velocity.

S2: Yes it can....it depends......

I: Can you elaborate.

S2: (Writes the equation $v = \omega r$)......ω is the angular velocity..... From the equation $\omega = v/r$. If v and r are not equal to zero, there is an angular velocity for the particle......

I: How does the angular velocity of the particle change as the particle moves......

S2: Is the velocity v increasing.

I: v is a constant.

S2: Then its simple..... angular velocity decreases........

I: Can you explain.

S2: From the same equation $\omega = v/r$......As the particle moves r increases...... (shows it by drawing a straight line, an origin and two rough lines from origin to two distinct points on the line of motion).......Since r is in denominator......angular velocity decreases.......

I: What about angular acceleration ?

S2: (Thinks for a moment).....If angular velocity is changing......yes there is an angular acceleration.....

I: Is there a torque acting on the particle.

S2: If there is an angular acceleration.....there should be a torque.......

I: Why do you say so?

S2: Because torque $\tau = I\alpha$........

I: Can there be a torque without a force.

S2: (Thinks for sometime).... No.....

I: Is there a force here on particle.....

S2: (Silent for sometime).....No.......but there has to be a torque........since there is an angular acceleration there should be a torque........

I: Why do you say so?

S2: It is like Newton's second law.......there can't be acceleration without a force........ I am confused........I don't know.......I am not sure......but I think there should be a torque...........

2.5.3 Development of items 26, 27 - Appendix A: Interview excerpts

Items 26 and 27 were developed to probe student understanding of angular velocity of a rigid body rotating about a fixed axis. The excerpts from a student interview given below helped develop aspects of these two items.

I: What do you mean by angular velocity of a rigid body?

S3: (Draws a circle and an axis through its center. Then draws a radius.) The object is rotating about the axis. Angular velocity $\omega = \Delta\theta/\Delta t$...... $\Delta\theta$ is the angle traced by this line (pointing to the radius and marking the angle).......

I: Let us draw an arbitrary rigid body, not a circle.

S3 Draws an arbitrary stone like shape and an axis through it.

I: What do you mean by the angular velocity of this rigid body you have drawn ? (Tells the student that a water bottle on the table may be used as a rigid body if that helps.)

S3: (Holds the water bottle in hand for some time, keeps it back and turns attention back to the drawing) It is complex......this rigid body has many particles.... I know $\omega = \Delta\theta/\Delta t$ ....but don't know which particle traces it........(Pauses for a moment).....One minute.... We will take the center of mass of the body.........

I: Why center of mass?

S3: Center of mass represents the whole body.......I remember using it while solving problems...... drawing the forces......all forces are considered to act on the center of mass........

I: OK. How do you find the angular velocity of the rigid body using its center of mass?

S3: (Marks a point close to the middle of the object as the center of mass). This is the center of mass (CoM)...... (pauses for sometime)...... this is the origin.......(marks a point O as origin outside the body in the figure, in the same horizontal plane as the center of mass, draws a line from O to CoM)..... This line traces an angle ......like this (pointing to the earlier figure of circle)...... I think this will be the angle......in $\omega = \Delta\theta/\Delta t$......but.......

I: Why that 'but' ?

S3: I think this is correct.....but.......this is not very clear...... (showing the figure and the $\Delta\theta$ marked in it)......the circle is very clear..........

We will discuss the salient aspects involved in the evolution of each of the 39 items of appendix A in chapters 3, 5 and 6. In these chapters we discuss the three parts constituting our inventory. The inventory which was developed systematically through the processes discussed so far was then validated and we discuss this next.

2.6 Validity of the Inventory

A good inventory should be able to measure what it purports to measure. The methodological construct which estimates how well the inventory does this is termed as the validity of the test. In psychometry one has different types of validities. A review of PER literature reveals that the usually employed validities with regard to concept inventories are face validity and content validity (Halloun and Hestenes, 1985; Beichner, 1994; Ding et al, 2006; Wuttiprom et al., 2009).

1. Face validity: Face validity is a prima facie assessment of the test and its appropriateness by the subjects (e.g., students who take test). The purpose is to ensure the clarity of statements of the questions and the distractors so that they are not misinterpreted (Adams et al, 2006). Our inventory was face validated by 10 practicing teachers, 10 higher secondary students and 8 doctoral students. Some of these candidates were part of our prior interactions/pilot studies mentioned in section 2.4.

2. Content validity: refers to assessing whether all relevant aspects of the concepts were adequately covered by the items. This is carried out by content experts. Our inventory was content validated by 8 experts which included senior professors and highly experienced undergraduate lecturers in physics. All of them had experience in designing various types of physics tests. They carefully analyzed each item and the corresponding distractors. We also carried out a semi-quantitative approach employed by Maloney et al (2001) which requires the experts to rate each item on a 5 point scale for reasonableness and appropriateness. Suggested modifications and changes were made to ensure that the inventory measures what it purports to measure. Suggestions given varied from minor modifications in wordings to dropping of the items. Some of the dropped items and a few illustrations of the suggested minor modifications are discussed in appendix C.

2.7 Administration of the inventory: Samples

The inventory was administered to four groups of students and two groups of teachers. We first discuss the student samples. In the first stage of administration the student groups comprised of 79 and 74 candidates respectively. All the students were at the higher secondary school level. The age range was 16-18. They had been taught rotational motion. We call the N=79 group as S1. They were from schools in Mumbai and it was a convenient sample. The percentage of boys and girls was almost equal in this sample. The group N = 74, which we denote as S2 were among the finalists who appeared for selection tests to represent India in the international olympiads in physics, chemistry and mathematics. The number of girls in S2 was 8 (11 %). The entire inventory was further administered to a large sample of over nine hundred students from 5 urban centers (Jaipur, Patna, Mumbai, Hyderabad and Bangalore) spread across the country. We denote this sample of 905 students as S3. The number of girls in this sample was around 360 (40 %). Requests were sent to schools in 7 urban centers and, among those who volunteered, 12 schools were selected. Our selection was influenced by (a) the geographical spread (b) variety in terms of certifying government boards (state vs central) and (c) administrative set up (private vs government schools). A part of the inventory was also administered to 384 introductory level students at the University of Washington, Seattle. We call this group as S4. A subset of 7 students from S2 and 35 students from S3 were interviewed.

The teachers taught physics at the HSS or undergraduate level. They were attending an exposure camp in physics olympiad in our institute and therefore constitute a convenient sample. The selection to the camp was done to ensure that (a) they were from across the country and (b) hailed mainly from semi-urban areas such as district towns and block level schools or colleges. The sample in the first stage consisted of 26 teachers. We denote the group as T1. A second group of 25 teachers (T2) participated in a later stage. Their demographics was similar to T1. The number of female teachers in both groups was around 30 %. Informal conversational interviews were carried out with a subset of 5 teachers each from T1 and T2.

2.8 Evaluation of the inventory

The performance in the inventory was gauged by assigning one mark to the correct answer. There was no penalty for wrong choices. No strict time restriction was imposed. Participants were asked to answer all items. Item level and whole test statistical analyses including item response curve analysis were carried out. The statistical indices calculated included difficulty level, index of discrimination, point biserial coefficient, Ferguson's delta and Kuder Richardson reliability index among others (Ding et al., 2006; Ding and Beichner, 2009). Difficulty level is defined as the ratio of the number of correct responses to the total number of students who attempted the item. The index of discrimination measures the extend to which an item can discriminate between low and high scoring students. The point biserial coefficient is a measure of correlation of students' score on the item and the score in the test. A high value indicates that students whose total scores are high are more likely to answer the item correctly. Ferguson's delta (δ) is a measure of the discriminatory power of the whole test. Kuder Richardson reliability (r_{test}) index measures the self consistency of the test. Item response curves (IRC) are a visually rich versatile tool for analyzing student responses. IRC involves a plot of the percentage of students $P_i(\theta)$ selecting a choice i to an item vis-a-vis their ability θ. Further details regarding these indices and techniques will be described in chapter 7 which is devoted to the analysis of the inventory.

Chapter 3

Rotational kinematics of a particle

3.1 Introduction

Rotational kinematics of a particle poses serious difficulties to students as well as teachers at the higher secondary school (HSS) level. The topic is often given short shrift by text books. After a brief discussion of angular velocity and and angular acceleration in the context of uniform circular motion, the concepts are directly introduced for rigid bodies. The pedagogical significance of a proper treatment of the single particle case, to ensure a smooth transition to the many-particle system or rigid body, is scarcely appreciated. These observations prompted us to investigate student difficulties in the topic and develop an inventory on the same (Mashood and Singh, 2012a). After the winnowing as described in chapter 2, this part of the inventory comprised of 18 items (items 1-7, 9-18 and 39). Another item was added later (item 8). The items are listed in appendix A.

In section 3.2 we discuss the development of the inventory for which we interacted with students as well as teachers. The validated inventory was administered to groups of students and teachers, details of which will be discussed in section 3.3. Here we also present our analysis of the collected data.

3.2 Content development of items

The evolution of items broadly followed the protocols described in chapter 2. In this section we discuss how the physics content of each item that constitutes the first part of the inventory developed. We describe how some of the observations during our interactions with students and teachers contributed to the development of questions/items and distractors. These questions are listed in appendix A. The numbering of questions in this chapter is in accordance with appendix A. Some students experienced trouble with ratios which was incorporated in question 1. Simple visualisations are required in basic rotational kinematics, often complemented by heuristic aids such as the right hand thumb rule. These aspects were addressed in question 2. The concept of limit or the instantaneous aspect associated with the definitions of $\vec{\omega}$ and $\vec{\alpha}$ presents difficulties to students. This is related to the non-discrimination between position and velocity where the students think that bodies moving in parallel have the same velocity when they reach the same position (Trowbridge and McDermott, 1980). Students tend to focus on the 'perceptually obvious phenomenon of passing' rather than the procedure for identifying instantaneous speed. Similar lack of discrimination between position and acceleration was also observed (Trowbridge and McDermott, 1981). Question 3 and its distractors were framed to probe rotational parallels to these pitfalls. The concept of limit was addressed partly in questions 7 and 14, besides question 3. The items 1-3 were posed in the familiar context of a wall clock. These questions pertained to the angular velocities of the tip of the second hand and the minute hand. The clock was made transparent to facilitate the probing of simple visualisations like how would the hands be moving when viewed from the rear.

We found that the direction of $\vec{\omega}$ was one of the most difficult things for the beginner students to comprehend. The notion that the vector ($\vec{\omega}$ or $\vec{\alpha}$) must be in the 'direction' of the actual motion of the particle is persistent. Even after repeated emphasis students reverted to the idea that $\vec{\omega}$ is 'in the plane of motion'. The phrase appears in the distractors to items 9 and 16. Observing this we emphasised to a group of students that $\vec{\omega}$ is 'perpendicular to the plane of motion'. A negative influence of this assertion was that some students started remembering the phrase like a cliche without understanding. The direction of $\vec{\omega}$ was probed in items 4 and

5. The contexts comprise of a particle in circular motion and a planet moving in elliptical path respectively. It may be noted that clarity regarding $\vec{\omega}$ is a prerequisite for correctly answering questions about $\vec{\alpha}$. The direction of angular acceleration was the theme of question 13, probed in the context of circular motion.

The angular velocity (like any other vector) can vary either with a change in magnitude or direction or both. We found that students ignore or forget one of the parameters. Often it was the directional aspect possibly because it is less familiar. Understanding and distinguishing both aspects of a vector clearly is important. We devised questions 9-11 to probe student understanding of variation in magnitude and direction of $\vec{\omega}$. An oscillating simple pendulum provided us with a rich context for this investigation. Similarly, understanding of the variation in magnitude and direction of angular acceleration was probed in questions 12, 16, 17 and 18. The confusion between linear velocity and linear acceleration had been documented in earlier works (Trowbridge and McDermott, 1981; Shaffer and McDermott, 2005). To investigate the existence of any rotational parallels, we decided to probe $\vec{\omega}$ and $\vec{\alpha}$ in the same context of a simple pendulum. With this aim items and distractors were framed almost identically for $\vec{\omega}$ (questions 7-11) and $\vec{\alpha}$ (questions 14-18). It may be noted that throughout the designing of the inventory we drew upon familiar and physically relevant contexts while posing the items.

A significant finding was that most students consider circular motion to be the prototype for rotational motion. Such fixations has its demerits which include recalling of memory fragments inappropriate to the situation. Another observation was the indiscriminate usage of equations ignoring the associated validity conditions, and this was the theme of question 6. We probed this issue by presenting elliptical motion to assess the understanding of the equation $\vec{v} = \vec{\omega} \times \vec{r}$, which in fact is valid only for circular motion. Probing a similar pitfall in the case of angular acceleration was the motive behind designing question 39 where the context was an accelerating giant wheel. We found the equation $\vec{a} = \vec{\alpha} \times \vec{r}$ was used ignoring the fact $\vec{a}$ here refers to the tangential component and not the total acceleration. It may be recalled that the conceptual dimensions probed by the inventory and the corresponding items probing them have been tabulated in chapter 2 (see table 2.1).

Table 3.1: The average performance of candidates of each group, the associated standard deviation and the average difficulty level. The corresponding percentage performances are in brackets. Total number of questions was 18 for students and 17 for teachers. Groups are described in chapter 2.

	S1 (N=79)	T1 (N=26)	S2 (N=74)
Average score	6.49 (36.06 %)	6.69 (39.35 %)	15.57 (86.50%)
Standard deviation	2.69	2.87	1.83
Average Difficulty Level	0.37	0.41	0.87

3.3 Administration and analysis

The inventory was administered to groups S1, S2 and T1, described in chapter 2. A subset of the students were interviewed. The average of the number of questions correctly answered by candidates of each group is given in table 3.1. The average performance in percentage is given in brackets. We also calculated the corresponding standard deviations and the average difficulty level (averaged over items) for all the three groups. The difficulty level (DL) of an item for a given group is defined as the ratio of the number of correct responses to the number of candidates who attempted the question. It may be noted from the definition that a more meaningful word for the index would be 'easiness level' (Ding et al, 2006).

The detailed response pattern of the student groups and the teachers is shown in table 3.2. The number of candidates responding to each of the choices of an item is displayed. We also calculated the difficulty level (DL) of each item for all the three groups. As can be inferred from table 3.2 the S2 group answered most of the items correctly. The response pattern of S1 as well as T1 exhibit pitfalls in understanding. In what follows we mainly discuss these two groups. The difficulty level index of most of the items pertaining to $\vec{\omega}$ are centred around 0.5. The teacher group scored slightly better for all items except question 6. For items on $\vec{\alpha}$ the difficulty level index drops for both groups. It may be recalled that magnitude wise difficulty level represents easiness. The value of DL is centred approximately around 0.35 for teachers as well as students. The DL index of some of the items was very low, particularly questions 17, 18 and 39. This prompted us to revisit the items and seek expert opinions regarding the questions. This iterative aspect involved in the construction of an inventory helps in refining the test as a diagnostic tool. In this particular case experts were satisfied with the content aspect of the questions. Simple modifications in the wordings to enhance simplicity and clarity were suggested.

Broadly, the results that can be inferred by an analysis of table 3.2 are coherent with what was observed in the pilot studies and associated interactions. Let us analyse the response pattern to some of the questions in greater detail. A perusal of the frequency with which distractors to each of the 18 items (see appendix A) were chosen reveals interesting patterns. This was confirmed later by interviewing a subset of students. Some of the pitfalls identified are similar to those found earlier in linear kinematics. While others are peculiar to rotational motion. Some difficulties overlap both domains, such as the concept of ratio. The concepts $\vec{v}$, $\vec{a}$, $\vec{\omega}$ and $\vec{\alpha}$ basically involve ratios. A qualitative as well as quantitative understanding of ratios is integral to understanding these concepts (Trowbridge and McDermott, 1980; Arons, 1990). As we can see from the response pattern to item 1, half of the candidates have problems in calculating and comparing simple ratios. For question 2 the options *b* and *c* which wrongly depicted the direction of $\vec{\omega}$ were chosen by a significant majority of students and teachers. This indicates inadequacies in visualisations as well as usage of heuristic aids like the right hand thumb rule. It also suggests that many people think that angular velocity of the tip of the clock (transparent) hand would change direction depending on whether we are looking at it from the front or the rear, since the motion changes from clockwise to counter clockwise.

Further analysis of the choice of distractors and the interviews helped us categorise the pitfalls and difficulties under the following broad themes.

3.3.1 Fixation with inappropriate prototypes

As mentioned earlier students had difficulty in comprehending the non intuitive direction of $\vec{\omega}$ and correspondingly $\vec{\alpha}$. This is validated by the number of teachers and students who wrongly chose *c* as the answer to question 4. The distractor *c* states that the direction of $\vec{\omega}$ of a particle in circular motion with increasing speed is different at two distinct points on the trajectory. One student who opted *c* explained that: *angular velocity tells about motion of objects in rotational motion ....in circular*

Table 3.2: Boldfaced numbers indicate the correct choice. The numbering of questions are in accordance with their position in appendix A. Question 39 was not administered to the teachers because it was in the developmental stage at the time of administration. Some of the participants left a few questions unanswered despite being requested otherwise.

	S1 (N=79)					T1 (N=26)					S2 (N=74)				
Q. No.	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL
1	16	14	**42**	7	.53	3	4	**13**	6	.50	0	1	**71**	1	.97
2	5	33	20	**21**	.27	6	5	7	**8**	.31	0	4	20	**50**	.68
3	**35**	6	26	12	.44	**13**	3	3	6	.52	**70**	2	0	2	.95
4	**29**	6	33	11	.37	**11**	1	8	3	.48	**68**	2	2	2	.92
5	19	11	27	**22**	.28	5	2	6	**11**	.46	0	1	2	**71**	.96
6	19	17	**32**	11	.41	11	7	**4**	3	.16	45	3	**25**	1	.34
7	**41**	9	8	21	.52	**17**	5	2	1	.68	**68**	5	0	1	.92
9	8	20	**25**	26	.32	3	4	**11**	7	.44	1	1	**63**	9	.85
10	20	3	12	**44**	.56	6	1	1	**18**	.69	1	0	1	**72**	1
11	**46**	13	16	4	.58	**16**	1	7	2	.62	**74**	0	0	0	1
12	18	11	**29**	21	.37	3	5	**9**	9	.35	2	1	**68**	3	.92
13	17	**32**	23	7	.41	7	**11**	5	3	.42	4	**65**	4	1	.88
14	25	10	10	**34**	.43	10	4	1	**10**	.40	1	7	0	**66**	.89
15	2	**29**	30	18	.37	2	**8**	9	6	.32	0	**69**	1	4	.93
16	7	22	22	**28**	.35	1	8	9	**7**	.28	0	0	4	**70**	.95
17	15	9	43	**12**	.15	7	0	13	**6**	.23	2	1	3	**68**	.92
18	40	10	19	**10**	.13	15	1	7	**2**	.08	10	0	1	**63**	.85
39	39	20	11	**9**	.11						14	6	3	**51**	.69

motion at different points they are moving in different directions. Further insights into the same difficulty can be obtained by looking into the response pattern to question 5. A majority wrongly chose the first three distractors, all of which implying that angular velocity lies in the plane of motion. This suggests that students are unable to grasp the idea that direction of $\vec{\omega}$ of a particle is always perpendicular to their position vector. It has been noted earlier that students' performance is impeded by 'fixation' to prototypical notions (Reif and Allen, 1992). What distinguishes the present case is that the prototype is a formally learnt one. The notion that a vector should be in the plane of the position vector is a 'hangover', particularly from linear kinematics and not a preconception acquired from everyday life. The difficulty with the direction of $\vec{\omega}$ will evidently carry over to $\vec{\alpha}$ and this is indicated by the response to items 12 and 13. Around 60 % of both students and teachers made wrong choices. Students are often unable to think beyond the circular motion framework when it comes to rotational motion. The following remark from a student illustrates this notion: *there are two types of motion.....linear motion and rotational motion......in linear motion objects move in straight line and in rotational motion they move in circle like wheel or a fan.....* We will report an extreme case of this 'fixation' where students and teachers were reluctant to ascribe rotational motion concepts to a particle in rectilinear motion (origin not on the path), in the next chapter (Mashood and Singh, 2012b).

3.3.2 Indiscriminate usage of equations

Responses to question 6 reveal indiscriminate usage of equation $\vec{v} = \vec{\omega}\times\vec{r}$, ignoring that it is valid only for circular motion. The context was a planet moving in an elliptical orbit. A majority erroneously chose *a* and *b* which says that the equation holds true either 'by definition' or 'because the planet is in rotational motion' respectively. A student noted: *the equation $\vec{v} = \vec{\omega}\times\vec{r}$ is always true..... it connects angular and linear velocity.* Surprisingly the students did far better in this question compared to the teachers. The corresponding pitfall for angular acceleration concerns the usage of $\vec{a} = \vec{\alpha}\times\vec{r}$, which was the theme of item 39. The students' performance was unsatisfactory, the difficulty level index dropping to its minimum. The most popular incorrect choice was *a*, chosen by about half of the students. As per the distractor the equation holds true 'because the motion is circular'. In fact the equation does not hold true in the case because $\vec{a}$ has a radial component as mentioned in the correct choice *d*. The item was not administered to the teachers being under development at the time of administration.

3.3.3 Pitfalls paralleling those found earlier in linear kinematics

Here we discuss those pitfalls which are similar in pattern to those identified earlier for linear velocity and linear acceleration. The response pattern to question 3 clearly shows that almost one-third of the students in S1 harbour the misconception that angular speed of the tip of the second and minute hands are same when their positions overlap. A student who made the choice said: *when they are both at 12'O clock their motion looks same.....both of them move in the same way for that small time interval...we can see it....* This is similar to the position-velocity confusion reported for one-dimensional motion (Trowbridge and McDermott, 1980; Hestenes and Wells, 1992). Prior work on linear kinematics documented a variety of confusions between $\vec{v}$ and $\vec{a}$ among students (Trowbridge and McDermott, 1981; Shaffer and McDermott, 2005). As mentioned before we devised items 7 to 11, 14 to 18 with the aim of ferreting out their rotational counterparts. The former set concerns angular velocity and the later set pertains to angular acceleration, both posed in the same context of an oscillating simple pendulum. In addition to uncovering interesting pitfalls concerning both the concepts we found an underlying pattern in students' reasoning of $\vec{\alpha}$. The phrase 'as $\vec{\omega}$ behaves so does $\vec{\alpha}$' succinctly captures the essence of the pattern, which is illustrated through what follows. A majority of the students and teachers correctly answered item 7 that angular velocity of the pendulum bob is zero at the extreme position. But a significant number of candidates thought that even the angular acceleration is zero at the point (item 14). The following remark by a student illustrates this: *At the extreme point, the pendulum is at rest for some time. It does not move. That means the bob has no velocity or acceleration at that point...for some time.* The misconception that when velocity is zero, acceleration at the point is also necessarily zero had been probed earlier in varying contexts (Reif and Allen, 1992; Hestenes et al., 1992; Shaffer and McDermott, 2005). The case of a ball thrown vertically up is a popular example. The students are asked about the velocity and acceleration at the point of maximum height. Our observation establishes that identical pitfalls exists in rotational kinematics as well. Questions 9 and 16 present another instance of candidates thinking that the angular acceleration would mimick angular velocity in its behaviour. The items pertain to the directions of $\vec{\omega}$ and $\vec{\alpha}$ as the pendulum swings from one extreme to the other. The correct choice to item 9 is *c* which says that the vector 'is perpendicular to the plane of motion and remains the same'. The same but incorrect choice was made for $\vec{\alpha}$ by a significant majority (item 16).

Questions 10 and 17 provide a third illustration of the pattern concerning $\vec{\alpha}$ mentioned above. More than half of the candidates rightly answered item 10 that ω first increases and then decreases as the pendulum bob moves from one extreme position to the other. However, when it came to question 17, which asked about the variation of α in the same context a majority of students as well as teachers wrongly chose *c*. The distractor *c* is identical in words to the correct choice of question 10, namely that 'it first increases and then decreases'. One student who made this choice explained: *Initially the pendulum bob speeds up as it start moving from left end. It comes to rest at the right end. So after the mid point speed will decrease. If speed is increasing that means acceleration is increasing.....if speed decreases acceleration decreases.* It is tempting to think that an increasing/decreasing angular velocity always implies a similarly varying angular acceleration. But as illustrated by the motion of the pendulum bob ω may increase while α is decreasing or vice versa. A related pitfall has been reported in the case of linear acceleration (Shaffer and McDermott, 2005). The last of our patterns indicating confusion between the behaviour of $\vec{\alpha}$ with that of $\vec{\omega}$ is revealed by responses to items 11 and 18. Question 11 probes the angular velocities of the pendulum bob at an intermediate point as it oscillates to and fro. About 60 % of the candidates correctly answered that angular velocities 'remain equal in magnitude but differ in direction'(choice *a*). The remark made by a student illustrates this reasoning: *When the pendulum oscillates back from right to left..... the orientation of motion changes....it is now anti-clockwise....so angular velocity and angular acceleration will change their direction....by right hand thumb rule.* The angular accelerations (item 18) however remain same in both magnitude and direction. The actual performance was not satisfactory. More than half the candidates wrongly chose *a* which once again revealed the existence of erroneous notion that $\vec{\alpha}$ always tracks $\vec{\omega}$.

3.4 Concluding remarks

The fact that there exists no significant difference between the performance of S1 group and the teachers is alarming. We have described the teacher group in section 2.7 of chapter 2. It may be recalled that the teachers were mainly from district towns and block level schools or colleges. Lack of meaningful teacher training programs in the country is one of the plausible reasons for their low scores. In addition the prevalent evaluation system is unimaginative and fosters rote learning exercises. Consequently teachers are pressurized into 'teaching to tests' that hardly assess conceptual knowledge or genuine problem solving skills. This over a period of time has blunted their abilities to tackle physics problems. When asked about the reason for their low scores some of the experienced teachers said that they could have answered these questions better in the earlier phases of their career. But now the pressure of teaching to tests that form the basis for grades and socio economic issues such as managing family, school and health have blunted their faculties for physics. Coming to implications for instruction, we have to accept that acquisition of even elementary concepts takes an extended period of time (Trowbridge and McDermott, 1980). It is important that the student be exposed to the same concept in a variety of physical contexts. Further, we suggest explicit emphasis on operational definitions and accompanying procedures. This has been repeatedly emphasised earlier, particularly in the case of linear kinematics (Arons, 1990; Reif and Allen, 1992; Shaffer and McDermott, 2005; Reif, 2008). Since one of the most important aspect of understanding rotational kinematics is proficiency with vectors, stressing operational definitions may help significantly.

Summarising, we find that rotational kinematics of a particle poses difficulties to higher secondary students as well as teachers. The topic has not yet received the attention it deserves from the PER community and is inadequately dealt with in most text books (Mashood and Singh, 2012c). We developed and administered an inventory on $\vec{\omega}$ and $\vec{\alpha}$ of a particle to groups of teachers as well as students. We described how items and distractors were constructed. The response pattern shown in table 3.2 was analysed. Areas of difficulties and probable pitfalls were identified. Some of the misconceptions uncovered parallel those reported earlier in linear kinematics. Instances corresponding to indiscriminate usages of equations were found. Fixations with prototypes resulting in errors were another significant observation.

Chapter 4

Rotational kinematics of a particle in rectilinear motion

4.1 Introduction

Motion of a particle along a straight line with constant velocity is the simplest situation in mechanics that one can envisage. One may however obtain pedagogically relevant insights if one consider its rotational kinematics about a point which does not lie on the path of motion (see figure 4.1). A student may be tempted to conclude that the concepts of rotational motion are irrelevant in this situation. Close and Heron (2011) have recently investigated the widespread tendency among students to ascribe zero angular momentum to particle moving in a straight line. This difficulty of associating angular momentum ($\vec{L}$) and related quantities with linear motion has been observed earlier (Palmieri and Strauch, 1963; Williamson et al., 2000). Physical intuition is not infallible (Singh, 2002). In this chapter we discuss the angular velocity of the particle ($\vec{\omega}$) and its variation as the particle moves from P, away from the origin O (see figure 4.1). The case represents an interesting situation where there exists an angular acceleration ($\vec{\alpha}$) despite zero torque ($\vec{\tau}$), which also implies that the often employed relation $\vec{\tau} = I\vec{\alpha}$ does not hold (here I is the particle's moment of inertia).

In section 4.2 we describe the rotational kinematics of a particle in rectilinear motion with constant velocity. It serves not only as a recapitulation for experts

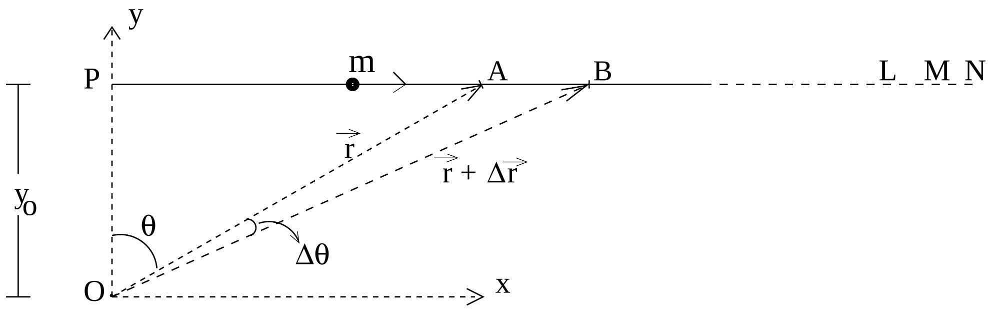

Figure 4.1: A particle of mass m is moving in a straight path PABN with constant speed V. The origin O is not on the path. The particle travels from A to B in time Δt.

but sets the tone for discussions in later sections. In section 4.3 we describe the content evolution of items constituting the inventory. The development was done with inputs from experts, teachers as well as novice students. The systematic and iterative character of the process of development have been described in chapter 2. Not only the students but even experienced teachers fell prey to some of the naive notions regarding angular velocity and angular acceleration of a particle in rectilinear motion. This was evident from the response patterns to diagnostic questions addressed to groups of higher secondary school students and a set of teachers. The administration and analysis of the inventory is discussed in section 4.4. Discussion of the pattern of errors and some immediate instructional implications are contained in section 4.5.

4.2 Rotational kinematics of a particle moving in a straight line

A particle in rectilinear motion has zero angular velocity only in the special case when the origin is located on the line of motion. As illustrated in figure 4.1, the magnitude of angular velocity of a particle is given by $\omega = \Delta\theta/\Delta t$ where $\Delta\theta$ is the angle traced in time Δt by the position vector of the particle with respect to the specified origin O . In other words, whenever the direction of the position vector of a particle changes there exists an angular velocity.

As the particle moves away from the origin ω starts decreasing. The decreasing angular speed can be inferred from the following consideration. From figure 4.1 we have

$$x = y_o tan\theta. \tag{4.1}$$

Differentiating Eq. (4.1) with respect to time and noting that $\dot{x} = V$ yields

$$\omega = V cos^2\theta / y_o, \tag{4.2}$$

where y_o is the perpendicular distance from the origin O to the line of motion. It may be noted that dot implies derivative. Substituting $cos\theta = y_o/r$ (here r is the distance of the particle from the origin) in Eq. (4.2) we obtain

$$\omega = V y_o / r^2, \tag{4.3}$$

which implies that ω decreases as the particle moves. It follows from Eq. (4.3) that

$$mr^2\omega = mV y_o, \tag{4.4}$$

a constant which is the magnitude of the angular momentum (L) of the particle. This point was made by Close and Heron (2011) also.

Let us take a brief look at the dynamics. A changing angular velocity implies an angular acceleration ($\alpha = \dot{\omega}$), but there is no torque! Zero linear acceleration and the corresponding absence of any force implies zero torque ($\vec{\tau} = \vec{r} \times \vec{F}$).

One can show the variation of the magnitude of angular acceleration by differentiating Eq. (4.2) with respect to time and employing Eq. (4.3), which yields

$$\alpha = -V^2 sin2\theta / r^2. \tag{4.5}$$

Thus in contrast to the relation that we often encounter in physics here we have a case where

$$\tau \neq I\alpha. \tag{4.6}$$

The reason is that for a particle moving in a straight line about a fixed origin, the moment of inertia is not constant but changes with time. Succinctly put, $\tau = I\alpha$

is not a basic equation. The more basic equation is $\tau = \dot{L}$ (Chabay and Sherwood, 2011). For the particle in our case it follows from Eq. (4.4) that $L = mr^2\omega$. When we associate $I = mr^2$ we obtain

$$\tau = I\alpha + \omega\dot{I}. \tag{4.7}$$

The rhs can be shown to be zero. Note the analogy with $F = \dot{p} = m\dot{V} + V\dot{m}$ (Newton's second law) when mass is not a constant.

4.3 Content development of items

The seemingly non-intuitive aspects of rectilinear motion outlined in section 4.2 prompted us to develop an inventory. In this section we provide some detail about how the interactions with students and teachers helped to frame and modify the questions as well as the distractors. We observed a tendency, mostly among novices, to reject outright the relevance of rotational motion concepts because the motion of the particle is in a straight line. As such, we incorporated this element in the distractors to all questions (the questions 19-25 are listed in appendix A). As an illustration of how distractors are modified after the pilot tests, consider question 19 where initially the distractors probed only relative magnitudes of angular speeds at two distinct points on the trajectory. We found from interviews that many students selected the correct choice but employed incorrect reasoning; they used the relation $v = \omega r$, which is valid only for circular motion. As a result of this interaction, we reframed the distractors so that this error was explicitly addressed.

Question 20 relates to our experience that the direction of angular velocity is one of the first hurdles that a student encounters when learning rotational motion. Question 21 probes the concept of angular acceleration. The distractors are deliberately restricted to kinematics so that this item is consistent with questions 19 and 20. The variation of angular acceleration dealt with in question 22 is a reasonably difficult topic even for experts; algebraic manipulations and/or asymptotic reasoning are necessary. Question 23 is intended to probe the relation between torque and angular acceleration. Most students strongly hold the view that an angular acceleration should always imply a torque. The influence of the analogy that acceleration is impossible without a net force (when mass remains constant) was evident. Another interesting finding was that people employed the invalid relation $\tau = I\alpha$ along with the wrong assumption that α is zero to arrive at the correct answer, namely, that torque is zero, to question 23. We modified the initial version of the distractors so that these pitfalls could be incorporated. Such an iterative process of interaction and modification helped refine the inventory and honed it as a diagnostic tool.

Question 24 can serve as a check for internal consistency because a correct response is meaningful only if some of the earlier items (e.g., 19, 20, and 21) are answered correctly. Question 25 probes student understanding of how the radial and azimuthal components of linear velocity change as the particle moves. A knowledge of the component (radial and azimuthal) aspects of linear velocity and acceleration is essential for a proper understanding of rotational kinematics. We emphasize once again that the protocols outlined in chapter 2 were followed in the development of items and distractors.

4.4 Administration and analysis

We administered the questions to two groups of HSS students and a set of 26 physics teachers. These were the same groups, S1 (N=79), S2 (N=74) and T1 discussed in the previous chapter. They are described in chapter 2. As one may recall, one of the two groups of students were the olympiad aspirants (S2). Table 4.1 gives the average of the number of questions correctly answered by candidates of each group. The average performance in percentage is given in brackets. We also calculated the corresponding standard deviations. As can be inferred from the average scores, the performances of the S1 and T1 groups reveal misconceptions and pitfalls. Even a section of S2 found the questions difficult as indicated by their drop in average relative to the performance described in the previous chapter. As mentioned in chapter 2 a subset of students were interviewed.

Despite being asked to answer all items, a few left some questions unanswered. As can be inferred from the low values of difficulty level, the questions were demanding. The choices corresponding to the naive notion that angular velocity and acceleration remain zero since the motion is linear are *d, d, a* and *a* respectively for the first four questions (19-22). The number of students (both groups) and teachers

Table 4.1: The average performance of candidates of each group and the associated standard deviation. The corresponding percentage performances are in brackets. Groups are described in chapter 2. Total number of questions was 7 for students and 5 for teachers.

	S1 (N=79)	T1 (N=26)	S2 (N=74)
Average	1.29 (18.43 %)	1.19 (23.8 %)	4.62 (66 %)
Standard deviation	1.44	1.49	1.70
Average Difficulty Level	0.19	0.24	0.67

Table 4.2: Boldfaced numbers indicate the correct choice. Questions 24 and 25 were not administered to the teachers because it was in the developmental stage at the time of administration. Some of the participants left a few questions unanswered despite being requested otherwise. The numbering of the questions is in accordance with appendix A.

	S1 (N=79)					T1 (N=26)					S2 (N=74)				
Q. No.	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL
19	22	**11**	15	31	.14	9	**5**	2	10	.19	4	**45**	2	21	.63
20	21	9	**27**	22	.34	1	5	**12**	8	.46	0	0	**72**	1	.99
21	43	11	**12**	13	.22	12	4	**4**	5	.17	23	0	**50**	0	.68
22	39	20	7	**13**	.16	14	3	5	**2**	.08	27	6	4	**36**	.49
23	**10**	21	29	19	.13	**8**	2	13	2	.32	**38**	18	17	1	.51
24	24	27	**11**	17	.14						22	3	**43**	6	.58
25	**18**	3	28	30	.23						**58**	7	5	4	.78

who chose them are significantly high as we can see from the table 4.2. The following statement from a student was typical of the perception of many students: *The particle is in linear motion. It is not rotating.... angular velocity....angular acceleration... are for rotational motion.* The surprisingly odd higher success rate for question 20 arises probably from mere recall of the often repeated phrase 'angular velocity is perpendicular to the plane of motion'. Another interesting observation is the number of students from S1 and teachers who chose *a* as the answer for item 19 wrongly assuming that the equation $v = \omega r$ is valid. A student who chose *a* explained: *The particle has an angular velocity. We have the equation* $v = \omega r$*. Angular velocity is given by* $\omega = v/r$ *.....*v *is constant and there is an* r *which is increasing.* Similarly for question 23 a significant percent from all groups chose *c* which is incorrect because the relation $\tau = I\alpha$ is not valid. A student from S2 correctly identified that $\tau = I\alpha$ is invalid because moment of inertia I is varying. She also said that: *I have learnt about changing moment of inertia in rigid body rotation... like a rotating dancer folding her arms....$\tau = I\alpha$ does not hold there.....also this is similar to rocket motion where mass varies as it moves.*

Distractor *a* for item 24 which say $\vec{\alpha}$ is undefined since its magnitude is zero was chosen by 30 % of both groups of students. One student stated: *The particle is moving in a straight line... It does not have any angular acceleration.....how can there be any direction for zero angular acceleration.* The response pattern to question 25 indicates that the popular distractors are *c* and *d* chosen by about 73 % of S1 (N=79). The distactors incorrectly states that one (radial) or both of the components of the velocity remains constant as the particle moves. The typical explanation given was: *the velocity of the particle is constant...it is given.....if the total velocity is constant, the components also remain constant*, as said by a student. The performance of S2 students to the item however is satisfactory with a DL of 0.78.

4.5 Concluding remarks

Incorrect reasoning based on recall of memory fragments or by comparison with an inappropriate prototypical situation has been reported in other contexts as well (Reif and Allen, 1992; Shaffer and McDermott, 2005). Emphasizing operational definitions and procedural specifications with the formal statements is an immediate corrective measure (Arons, 1990; Shaffer and McDermott, 2005; Reif, 2008). Addressing specifically the problem of angular velocity of a point particle we suggest the following operational definition.

A particle has an angular velocity about an origin if the position vector of the particle with respect to the same origin changes in direction with time.

Procedurally

- Draw the position vector $\vec{r}$(t) of the particle with respect to a specified origin at some instant t.

- Draw the position vector $\vec{r}$(t+dt) of the particle with respect to the same origin after some time, say t+dt.
- If $\vec{r}$(t) and $\vec{r}$(t+dt) differ in direction then the particle has an angular velocity.

The distractors to an item were deliberately designed to create cognitive conflicts so that people would employ more than one argument for verification. Nevertheless, except for a few experts, candidates rarely checked the consistency by arguing in different ways. Expert argumentation at times even generated interesting and novel approaches. For example, in question 19 one of the experts argued that: *straight line motion is a special case of planetary motion with zero central force. From Kepler's second law, equal areas ($r^2\Delta\theta/2$) are swept in equal times Δt. Thus $r^2\omega$ is a constant and therefore ω should keep decreasing.* The ability to identify structural similarity between problems is an expert characteristic that ensures efficient problem solving (Singh, 2008).

Another interesting observation was the visual appraisal of problems by experts. Most of them answered item 19 by simple inspection of the figure; in fixed time Δt, $\Delta\theta$ decreases as the particle moves away from P. Deployment of asymptotic reasoning was another observed striking expert characteristic. Question 25 was answered by inspection of the motion at extreme points P and N. At P the radial component of velocity is zero and the tangential component is maximum. At infinity the tangential component vanishes and the velocity is purely radial. Asymptotic arguments were employed in answering questions 19 and 22 too. It was argued that at large r the angle traced by the position vector of the particle will be small, decreasing for an interval of time.

Chapter 5

Variation of angular velocity and angular acceleration of a particle in rectilinear motion

5.1 Introduction

In this chapter we discuss the angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$) associated with a particle in rectilinear motion with constant acceleration. The work was motivated by an observation that students and even teachers have difficulty in ascribing rotational motion concepts to a particle when the trajectory is a straight line as discussed in chapter 4. Contrary to (naive) expectations, the particle possesses an angular velocity($\vec{\omega}$), angular acceleration ($\vec{\alpha}$) and angular momentum ($\vec{L}$) if one considers the origin outside the line of motion (see figure 5.1). A formal derivation of ω and α is presented which reveals 'surprising' and non intuitive aspects, namely non monotonous behavior with associated extremum. The special case of constant velocity is studied and we find that the angular acceleration associated with it also has an extremum.

In section 5.2 we discuss the variation of angular velocity as the particle moves away from A. We present a formal derivation and also numerically trace ω. While it is possible to visualize the fall in ω with r (distance of the particle from the origin) for the constant velocity case, the maximum in ω for constant acceleration (a) comes as a surprise. In section 5.3 we discuss the variation of α. Once again numerical plots are preceded by formal results. Here one is confronted with non monotonic behavior for both cases, zero and constant acceleration. Thus a seemingly monotonous system as a particle in straight line motion exhibits surprising features.

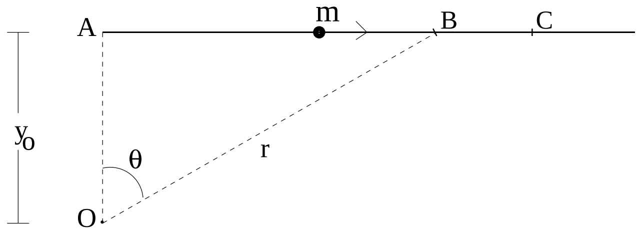

Figure 5.1: A particle of mass m is moving in a straight path ABC. The origin O is not on the path. B and C are two distinct points on the trajectory. The perpendicular distance from origin to the line of motion is y_o.

5.2 Variation of angular velocity

The position vector of the particle is given by

$$\vec{r} = r\hat{r}. \tag{5.1}$$

Differentiating equation (5.1) with respect to time we obtain the velocity,

$$\vec{v} = \dot{r}\hat{r} + \omega r\hat{\theta}, \tag{5.2}$$

as $\dot{\hat{r}} = \omega\hat{\theta}$ where $\hat{r}$ is the radial unit vector and $\hat{\theta}$ is the tangential unit vector. It may be noted that dot denotes differentiation.

The angular momentum of the particle at any instant is

$$\vec{L} = \vec{r} \times (m\vec{v}), \tag{5.3}$$

which directly yields $L = mvy_o$. Equation (5.2) in equation (5.3) yields $L = mr^2\omega$. Comparing the two expressions for the magnitude of angular momentum we obtain the variation of ω with r,

$$\omega = vy_o/r^2. \tag{5.4}$$

Note that the angular velocity of a particle depends on the choice of origin. When $y_o = 0$ we have $\omega = 0$.

To express ω as a function of single variable r consider the equation of motion

$$v^2 = {v_o}^2 + 2ax, \tag{5.5}$$

where v_o is the velocity of the particle at A (at time $t = 0$) and x is the distance traveled in time t.

Equation (5.5) in equation (5.4) along with $x = \sqrt{r^2 - {y_o}^2}$ yields

$$\omega = \frac{y_o\sqrt{{v_o}^2 + 2a\sqrt{r^2 - {y_o}^2}}}{r^2}, \tag{5.6}$$

which shows the variation of ω with r of a particle moving with constant acceleration. As $r{\rightarrow}y_o$, ω approaches v_o/y_o. At infinity ω vanishes as $r^{-3/2}$.

To obtain the variation of ω for the constant velocity case insert $a = 0$ in equation (5.6). The equation reduces to

$$\omega = y_o v_o/r^2. \tag{5.7}$$

We can see from equation (5.7) that ω decreases monotonously as $1/r^2$ as shown in the inset of figure 5.2. It may be noted that we have taken unit values for y_o, v_o and a (i.e. $y_o{=}v_o{=}a{=}1$ in numerical terms) while plotting all the graphs and calculating extrema. The decrease in ω can also be inferred by visual inspection of figure 5.1. Since the velocity of the particle is a constant the angle traced by the position vector in equal intervals of time keeps decreasing as the particle moves away. Yet another approach to understanding the behavior is by asymptotic arguments. At A the linear velocity ($\vec{v}$) is purely tangential and thereby angular velocity is maximum while at infinity $\vec{v}$ is purely radial and ω vanishes.

But when the particle is moving with a constant acceleration ω increases initially and then decreases as shown in figure 5.2. The behavior is non intuitive. That the

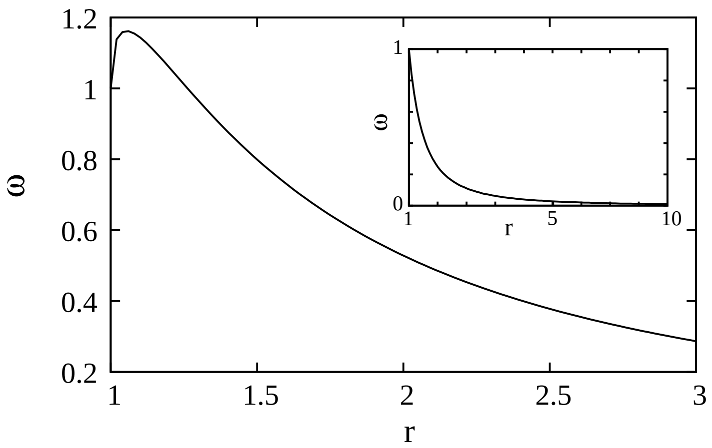

Figure 5.2: Variation of ω with r when the particle is moving with constant acceleration. The inset shows the special case of constant velocity. See text for details.

angular variables will vanish at infinity can be understood by the fact that the motion will approach linearity as the distance from origin increases. In the next section we argue the existence of an extremum in ω, but meanwhile we obtain this maximum by differentiating equation (5.6) with respect to r which yields

$$3r^2 + 2\sqrt{r^2 - 1} - 4 = 0 \tag{5.8}$$

To solve the equation make the substitution $r^2 - 1 = x^2$ where $x > 0$, which yields the maximum at $r = \sqrt{10}/3$.

5.3 Variation of angular acceleration

The variation of angular acceleration is obtained by differentiating equation (5.6) with respect to time. We get $\dot{r}$ in terms of r by squaring equation (5.2) and substituting for v from equation (5.4), which implies

$$\dot{r} = (\omega r\sqrt{r^2 - {y_o}^2})/y_o. \tag{5.9}$$

The magnitude of angular acceleration for the case of constant linear acceleration is thus obtained as

$$\alpha = \frac{ay_o(4{y_o}^2/r^2 - 3)}{r^2} - \frac{2y_o{v_o}^2\sqrt{r^2 - {y_o}^2}}{r^4}. \tag{5.10}$$

When $a = 0$ we have

$$\alpha = \frac{-2y_o{v_o}^2\sqrt{r^2 - {y_o}^2}}{r^4}, \tag{5.11}$$

which is the variation of angular acceleration of a particle in rectilinear motion with constant velocity.

Angular acceleration α initially decreases with r and then increases in cases of both constant velocity (see inset) and constant acceleration as shown in figure 5.3. Like the non monotonous behavior of ω in the previous section the variation in α is not obvious. Nevertheless, one can get some insights by visual inspection and asymptotic arguments. For the constant velocity case when $r = y_o$ we have $\alpha = 0$ (see equation (5.11)). At large r, α is negative and approaches zero as $1/r^3$. Thus α will possess a minimum and this minimum can be obtained by taking $d\alpha/dr = 0$ in equation (5.11). As mentioned earlier we take y_o=v_o=a=1 in numerical terms. The minima lies at $r = 2/\sqrt{3}$ which gives $\theta = \pi/6$ rad. The case of particle moving with constant velocity also presents us with a simple context to introduce the concept of angular jerk, which is obtained by differentiating equation (5.11) with respect to time (Tan and Edwards, 2011).

We also note the asymptotic behavior of α when a is a constant. As $r \to y_o$, an inspection of equation (5.10) suggests that α is positive. One can also verify by substitution that $\alpha = 0$ at $r = \sqrt{10/3}$. This helps us understand the behaviour of ω in figure 5.2. For $r \in [y_o = 1, \sqrt{10/3}]$, ω increases and thereafter it decreases. Also note that as $r \to \infty$, α approaches zero from the negative side.

The minimum for the constant acceleration case is obtained by taking $d\alpha/dr = 0$ in equation (5.10), which yields

$$3r^2\sqrt{r^2 - 1} + 3r^2 - 8\sqrt{r^2 - 1} - 4 = 0 \tag{5.12}$$

The real solution for above cubic equation can be readily obtained by the substitution $r^2 - 1 = x^2$ where $x > 0$. The solution is $x = 1$ which implies that the point of minima is at $r = \sqrt{2}$. The corresponding value of θ is $\pi/4$ rad.

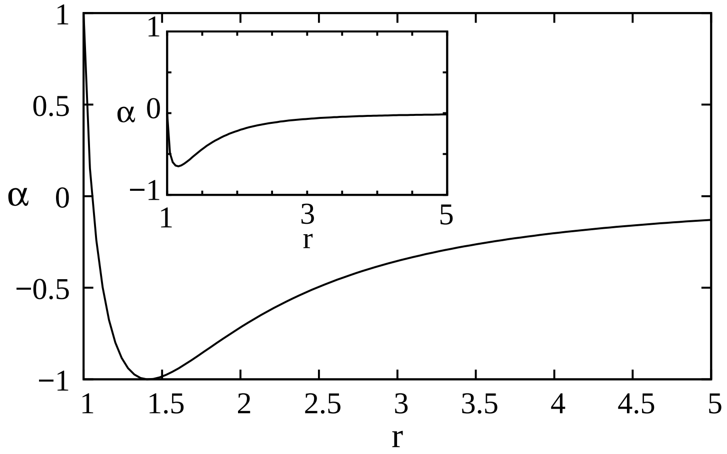

Figure 5.3: Variation of α with r when the particle is moving with constant acceleration. The inset shows the special case of constant velocity. See text for details.

5.4 Concluding remarks

A physical context for the problem discussed in this chapter can easily be constructed. Consider a vehicle moving on a straight horizontal road at dusk. A person standing at some distance away from the road can switch on her laser flashlight and follow the trajectory of this vehicle. The angular speed of the light beam would decrease monotonously with time. In contrast, were the flashlight turned skywards, the beam would have a constant angular speed if the person follows the trajectory of a satellite orbiting with constant speed. The switch from a straight road to a circular trajectory is responsible for the change from a monotonously decreasing angular speed to a constant angular speed.

Summarizing, contrary to common misconceptions, a particle in rectilinear motion has an angular velocity, angular acceleration and angular momentum if we chose the origin outside the line of motion. Angular velocity decreases monotonously as the inverse square of the distance from the origin when the particle is moving with a constant velocity. However, when the particle has a constant acceleration, ω initially increases and then decreases. The point of maximum lies at $r = \sqrt{10}/3$. Whereas angular acceleration of the particle decreases initially and then increases for particle moving with both zero and constant linear acceleration. The minimum in the constant velocity case lies at $r = 2/\sqrt{3}$ which gives $\theta = \pi/6$ rad. While in the constant acceleration case the minimum shifts further to $r = \sqrt{2}$ or $\theta = \pi/4$ rad.

Chapter 6

Rotational kinematics of a rigid body about a fixed axis

6.1 Introduction

As discussed in chapters 3 and 4, we carried out studies to probe students as well as teachers understanding of angular velocity and angular acceleration of a particle. We identified misconceptions and pitfalls in reasoning (Mashood and Singh, 2012a, 2012b). Some of them parallel those found earlier in linear kinematics while others were peculiar to rotational motion. The present investigation on rotational kinematics of a rigid body is a natural progression of our earlier work on a single particle.

In this chapter we present the development, administration and analysis of the third part of our inventory on rotational kinematics of a rigid body about a fixed axis. This part of the inventory comprises of 13 items which are listed in appendix A (items 26-38). In section 6.2 we discuss the content development of these items. This is followed by a discussion of administration and analysis in section 6.3. The systematically and iteratively constructed questions were validated and administered to a set of teachers T2 (N = 25) and two groups of pre-university students S2 (N = 74) and S3 (N=905) in India. The samples are described in chapter 2. A subset of the students were interviewed. Students, as well as teachers, exhibit difficulties in applying the operational definition of the angular velocity to a rigid body. Many erroneously assume that an angular acceleration cannot exist without a net torque. Patterns of reasoning resulting in errors were identified and categorized under broader themes. The inventory was also administered to introductory level students, S4 (N = 384) at the University of Washington. Popular distractors to most items remained similar to the Indian students. It may be recalled that the terms items and questions are used interchangeably.

6.2 Content development of items

The general protocols followed in the construction of items and distractors have been described in chapter 2. In this section we discuss how our interactions with students and teachers contributed to the content development of the items (26-38 listed in appendix A). During this study we observed that many students had difficulty with the concept of the angular velocity of a rigid body. Questions 26 and 27 were devised to probe student understanding of the operational definition of angular velocity. Item 26 was designed to investigate whether students could identify the angle $\Delta\theta$ in the definition $\omega = \Delta\theta/\Delta t$. Although the students were familiar with the idea of an angle being traced in case of a single particle, some seemed confused in the case of rigid body motion. We found a tendency among students to reduce the analysis to that of a single particle. Often this particle was the center of mass. The notion that the center of mass acts like a representative point for a rigid body is often useful in physics. However, this notion, along with the idea that an angle is traced by position vector of some particle, led many to incorrectly think that $\Delta\theta$ in $\omega = \Delta\theta/\Delta t$ is the angle traced by the position vector of the center of mass. We incorporated this notion as a distractor for question 26. Another related but erroneous idea was that $\Delta\theta$ is traced by the position vector of any particle with respect to a specified origin, which led to a second distractor to the first question. In the development of question 26 we also identified a tendency of students to treat the angular velocity of a body as the sum of the angular velocities of the parts (similar to how torque, angular momentum and rotational kinetic energy etc. of a rigid body is obtained by summing the respective quantity over all particles). This error was probed by question 27, in the context of a rotating pulley.

Simple visualizations are integral to understanding rotational motion. Question

28 probes student understanding of the trajectory of an arbitrary particle lying on a rigid body rotating about a fixed axis. In addition to the correct answer ('circle with center on the axis'), the distractors included 'spiral' as well as 'some complex curve'. As in question 26, we noted another variant of the misconception associated with the notion of the center of mass. Some students seem to think that the trajectory of a particle is circular only when the axis passes through the center of mass. This idea appears as the fourth distractor to the question. The point that angular velocity is the same for all particles on a rotating rigid body is very important. Nevertheless our interactions revealed that some students think angular velocity is distinct for each particle similar to its linear counterpart for a rotating body. The latter idea constitutes the subject of questions 29 and 30. We also observed as we developed the inventory that some students think that a particle closer to the axis moves faster than a particle far from the axis. The closer particle thus has greater linear and angular speeds. Some of the distractors for question 30 incorporated these aspects of student reasoning.

As described in chapter 3, we found that students had considerable difficulty in understanding the direction of angular velocity and angular acceleration of a particle (Mashood and Singh, 2012a). The same difficulty was observed in the case of a rigid body. We probed this partly through question 31 which also examines understanding of the variation of the magnitude and the direction of the angular velocity of a rigid body. Question 32 probes student thinking about the direction of the angular acceleration. Specifically, we investigated how the directions of the angular velocity and the angular acceleration are related when the former is decreasing. The potter's wheel, which is a familiar artifact in India serves as the context for the items 30, 31 and 32.

We discussed in chapter 4 that students find the idea that an angular acceleration can exist despite zero torque surprising (for a particle in rectilinear motion). Many were convinced by a kinematic analysis that an angular acceleration exists in case of the particle in rectilinear motion with constant velocity when the origin is not on the path. Simultaneously they also knew that the torque is zero as there is no force. A cognitive conflict ensued as to whether an angular acceleration existed. We observed a similar confusion in the case of rigid bodies as well. Item 33 addresses this confusion. One distractor incorporates a case of indiscriminate usage of the equation $\tau = I\alpha$ as well (Mashood and Singh, 2012b). Most students are unaware of the fact that this equation is invalid if the moment of inertia varies with time. Another important distinction that has to be kept in mind regarding angular velocity of a particle and a rigid body is the origin dependence, which is the theme of question 34. While the angular velocity of a particle varies with the origin, that of a rigid body is independent of any specified origin. The concept of origin is more or less insignificant as far as rigid body is concerned. Here the axis of rotation plays a key role.

A poorly differentiated understanding of linear kinematics was apparent in our interactions with the students. This finding has been repeatedly documented (Trowbridge and McDermott, 1980, 1981; Hestenes et al., 1992; Reif and Allen, 1992; Shaffer and McDermott, 2005). We consider here a few aspects that are relevant to rotational kinematics, such as student understanding of the relation between linear and angular kinematic variables. Some students think that linear acceleration ($\vec{a}$) and angular acceleration ($\vec{\alpha}$) are independent of each other. We incorporated this incorrect idea into items 35 and 36. The students who knew that these quantities are related based their understanding on the equation $a = \alpha r$ (here r is the radius). However many of them seemed to be unaware of the fact that a in the equation refers only to the tangential component of the acceleration. Questions 35 and 36 address these ideas as well. Items 37 and 38 were designed to probe student understanding of the components of linear acceleration, namely centripetal and tangential acceleration. Question 37 was posed in the context of a spinning wheel and question 38 in the context of a rotating giant wheel. The content of item 38 has been explored earlier in the context of a simple pendulum (Reif and Allen, 1992). All the items were validated as described in chapter 2. Face validity was carried out by students ensuring that the questions are interpreted as intended. Content validity was carried out by faculty who taught physics at the university level.

6.3 Administration and analysis

The validated items were administered to groups T2, S2 and S3. We discuss the administration to the S4 group in the next section. As mentioned earlier an item was gauged by awarding one mark to the correct choice. There were no negative marks for wrong choices. Participants were asked to attempt all the questions. Items 35, 36 and 37 were not administered to the T2 since the questions were in the developmental stage at the time of administration. Items 33 and 34 were not posed to S2 for the same reason. Table 6.1 shows the average performance, associated standard deviations and the average difficulty level for all the three groups. Similar to the response to part one of our inventory average score and the difficulty level of S2 is high indicating expertise. As such the focus of our analysis in what follows will be mainly on T2 and S3. In the remainder of our discussion, student refers to S3 unless mentioned otherwise.

Table 6.1: The average performance of each group along with their corresponding standard deviation and average difficulty level. Performance in percentages are given in parentheses. Total number of questions was 10 for T2, 11 for S2 and 13 for S3.

	T2 (N=25)	S2 (N=74)	S3 (N=905)
Average score	3.08 (30.80 %)	9.72 (88.36 %)	6.28 (48.1 %)
Standard deviation	1.75	1.37	3.87
Average difficulty level	0.31	0.88	0.48

Table 6.2 shows the number of candidates who chose each of the four choices for every question. From the responses we calculated a difficulty level (DL) for each item for all the groups. It may be recalled that the DL index is defined as the ratio of the number of correct responses to the total number of responses for an item. An analysis of the DL indices given in table 6.2 shows that the performance of the S2 group is significantly better. All items apart from questions 26 and 38 had a difficulty level of about 0.9. This indicates that this group of students possesses expertise in the topic. Analysis of the detailed response patterns of T2 and S3 based on the frequency with which distractors to each items were chosen indicate pitfalls and reasoning errors. The DL indices for the teachers are mostly between .25 and .55. There are also cases of extreme concern with the DL dropping to less than 0.1 (items 26 and 32). Further analyses of the response pattern of S3, aided by insights from interviews indicate that most of them could be categorized under the following four broad themes:

Table 6.2: Boldfaced numbers indicate the correct choice. The questions are listed in appendix A. Blank rows indicate questions which were not administered to the group since the questions were in the developmental stage. Some of the participants left a few questions unanswered despite being requested to do otherwise.

	T2(N = 25)					S2 (N=74)					S3 (N=905)				
Q. No.	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL	*a*	*b*	*c*	*d*	DL
26	6	16	**2**	1	.08	9	27	**34**	4	.46	159	350	**271**	101	.31
27	6	0	**9**	10	.36	1	0	**71**	2	.96	171	106	**457**	151	.52
28	**14**	2	2	7	.56	**68**	0	0	6	.92	**526**	78	101	186	.59
29	2	**14**	4	5	.56	1	70	2	1	.95	99	**529**	146	136	.58
30	5	7	2	**11**	.44	0	1	4	**69**	.93	206	151	104	**436**	.48
31	**7**	2	12	4	.28	**68**	2	2	2	.92	**507**	145	168	74	.57
32	12	**2**	6	5	.08	1	**71**	2	0	.96	171	**439**	182	98	.49
33	7	2	8	**9**	.32						137	114	320	**316**	.36
34	**6**	9	7	3	.24						**333**	164	255	127	.38
35						**68**	4	2	0	.92	**567**	110	115	114	.62
36						2	**70**	0	2	.95	242	**423**	94	135	.47
37						0	**71**	2	1	.96	90	**504**	240	69	.56
38	2	15	**3**	4	.17	0	11	**59**	4	.80	95	352	**374**	67	.42

6.3.1 Inappropriate extension of familiar procedural practices

Repeated application of procedural practices in a topic can result in students becoming habituated to them. This acquired familiarity can at times lead to their extending these procedures to contexts in which they are inappropriate. In the present study on rigid bodies, such practices can be traced to basic mechanics or rotational kinematics of a particle. For example consider the operational definition for $\vec{\omega}$ of a rigid body. The identification of $\vec{\omega}$ in the case of a single point particle involves the angle $\Delta\theta$ traced by its position vector. Many students seem to think that $\vec{\omega}$ for a rigid body is identified similarly. For instance, one student said: *An angle is always traced by a position vector. For rigid body also I think it would be similar.* The distractor *b* to item 26 incorporated this notion. It states that $\Delta\theta$ in the case of $\vec{\omega}$ of a rigid body rotating about a fixed axis is traced by the position vector of any particle on the body with respect to a specified origin. This answer was chosen by around 39% of the students. The correct answer, choice *c*, is that $\Delta\theta$ is the angle traced by a line perpendicular to the axis from any particle. Another instance of an inappropriate extension of familiar procedure is illustrated by the responses to item 27. This question pertains to the identification of the angular velocity of a rotating pulley. The incorrect choice *a* was selected by 19%percentage of students. This choice states that $\vec{\omega}$ of the pulley is 'the vector sum of the angular velocities of all the particles constituting the pulley'. A student who chose *a* noted: *To obtain momentum, kinetic energy, moment of inertia, angular momentum etc. of rigid body, sum of all particles is taken....isn't it..... Now every particle has an angular velocity..... We should take the sum of all of them.* The practice of summing a quantity over all particles is appropriate only for dynamical quantities like energy, momentum, rotational kinetic energy, angular momentum etc. Extending it to angular velocity is inappropriate. This tendency to extend a familiar practice to an unfamiliar or inappropriate situation is perhaps a variant of 'conceptual minimalism' that characterizes the thinking of novice students (Close and Heron, 2011).

6.3.2 Reasoning cued by primitive elements in thought

Concepts or ideas which resonate with more primitive ideas can influence our reasoning. Close and Heron (2011) describe an example of such a case where 'energy conservation appears to take a higher status in student thinking over other conservation laws'. We have independently observed instances of the same among Indian students. In our present study on rigid body motion we found a similar pattern wherein students ascribe a special status to the concept of center of mass (CoM). A student remarked: *CoM of a body can be at a point where there is no mass... I find this very interesting.....It was surprising when I first learnt it.....* Students extrapolate the notion of CoM as a representative point beyond what is warranted by physics. The special status of CoM seems to have influenced their answers to our questions. For item 26 the second most widely chosen distractor by all the three groups is *a* (the option that incorrectly states that $\Delta\theta$ appearing in $\vec{\omega}$ of a rigid body is the angle traced by the position vector of the center of mass of the body). About 18 % of the students made this choice. One of the students who made this choice said: *The angle has to be traced by some position vector. Only CoM represents the* *whole body.* The choice could also have been made to circumvent the difficulty in analyzing a rigid body by reducing it to a more comfortable choice of a single particle (CoM) (Ortiz et al., 2005).

The more striking case is question 27. Here one can easily see that the center of mass of the pulley is its geometric center and it is at rest. Nonetheless, around 17 % of the students stated that the angular velocity of the pulley is equal to the angular velocity of its CoM. Another instance of the apparent cuing influence of the concept of CoM is given by the response pattern to question 28. The most popular distractor is *c*, which states that the trajectory of an arbitrary particle on a rotating rigid body is circular only if the axis 'passes through the center of mass'. About 11 % of the students chose this option. One student gave the following argument for her choice: *The trajectory.....to be circular....everything has to be symmetric....that is possible only when axis passes through CoM.*

6.3.3 Lack of differentiation between related but distinct concepts

Novice thinking is often characterized by a failure to differentiate between related but distinct concepts. The case of velocity ($\vec{v}$) and acceleration ($\vec{a}$) is a well-documented illustrative example (Trowbridge and McDermott, 1981; Shaffer and McDermott, 2005). In the case of rigid body motion we find similar confusion between $\vec{v}$ and $\vec{\omega}$. Responses to questions 29 and 30 indicate that a significant majority of students fail to understand rigid body rotation in terms of the constituent particles. On question 29, the distractor chosen by around 16 % of the students was choice *c* which states that all particles on a rotating ceiling fan have the same $\vec{v}$ as well as the same $\vec{\omega}$. Their arguments were highly incoherent and confused. They do not realize that only the angular velocity is same for all the particles as encapsulated by the correct choice *b*. Distractor *d* to question 29 which conveys the idea that different particles have different $\vec{v}$ and $\vec{\omega}$ was also chosen by a considerable fraction of students (15 %). For question 30 the most popular distractors are *a* and *b* (the angular velocity of a particle closer to the axis is greater and vice versa, respectively). These choices along with the previously described distractor *d* to question 29 indicate that many think that each particle on a rotating rigid body has a distinct $\vec{\omega}$ like $\vec{v}$.

The following explanation from a student illustrates this notion: *Angular velocity is another velocity like linear velocity....it is used when body is rotating....It shows which way particle moves as body rotates and how fast... All particles on the body are different....each will have its angular velocity.* The student conception that angular velocity has characteristics similar to linear velocity has been pointed out previously in chapter 3 (Mashood and Singh, 2012a). A notable example is the persistent notion that direction of $\vec{\omega}$ of a particle is in the plane of motion, like $\vec{v}$.

Around 19 % of the students chose *c* for question 31. These distractors include the ideas that both the magnitude and direction of $\vec{\omega}$ of a slowing down potter's wheel change with time. In fact this is true only for $\vec{v}$ of each particle. In question 32 the most popular incorrect choice was *c* chosen by 20 % of the students. This distractor states that the direction of the angular acceleration of the potter's wheel is perpendicular to the axis of rotation. Note that it is the linear accelerations of particles constituting the rigid body which lie in planes perpendicular to the axis of rotation. Questions 31 and 32 thus reveals confusion about $\vec{\omega}$. A closer look at the erroneous distractors to these items discussed above indicates a tendency to ascribe aspects of $\vec{v}$ to $\vec{\omega}$.

6.3.4 Indiscriminate use of equations

Often students chose to rely on equations when confronted by a physics problem. This approach can be efficient provided the student is aware of the validity conditions associated with the equation at hand. Novices however often employ equations indiscriminately (Reif and Allen, 1992). An interesting illustration is given by question 33 which provides a case wherein there exists an angular acceleration without a torque. About 35 % of the students incorrectly answered this question relying on the relation $\vec{\tau} = I\vec{\alpha}$ (here I denotes the moment of inertia). As indicated by the frequency with which students chose distractor *c* many think that a 'torque acts on the particle resulting in angular acceleration as per $\vec{\tau} = I\vec{\alpha}$'. One student who opted *c* said: *If angular acceleration is there ...there will be a torque... it follows from* $\tau = I\alpha$. The fact that the spinning girl folding her arms represents a case of varying I and that the above equation holds only when I is a constant was not appreciated.

Items 35 and 36 give another illustration of indiscriminate use of equations.

About 63 % of the students correctly answered that the tangential acceleration remains constant for a particle on a wheel rotating with constant $\vec{\alpha}$. During our interactions we observed that the relation between $\vec{a}$ and $\vec{\alpha}$ is not well understood by students. Many tend to resort to the relation $a = \alpha r$ (here r is the radius). They often do not recognize that the term 'a' in the relation stands only for the tangential component of $\vec{a}$. On question 36, the most popular choice is *a* similar to question 35. This distractor incorrectly states that the centripetal acceleration is also constant for the particle under consideration. The following explanation from a student illustrates this further as a case of indiscriminate use of equations: *Centripetal acceleration is constant because $a = \alpha r$.....for the wheel both r and α are constants.*

Some of the difficulties we observed in the study could not be incorporated into any of the above four themes. In addition to the errors that can be attributed to the indiscriminate use of equations that relate $\vec{a}$ and $\vec{\alpha}$, we noted the following error. A significant majority, as indicated by the choice of distractor *d* to both questions 35 and 36, seemed to think that $\vec{a}$ and $\vec{\alpha}$ are unrelated. Their responses to question 37 reveal another incorrect idea. Many students seemed to think that both the tangential and the centripetal acceleration exist for a particle on a wheel moving with zero angular acceleration. The corresponding distractor *c* was chosen by about 27 % of the students.

Finally we address the issue of the origin dependence of $\vec{\omega}$ of a particle and a rigid body that was probed in question 34. The origin dependence of $\vec{\omega}$ of a particle was illustrated previously, chapter 4, in the context of a particle in rectilinear motion (Mashood and Singh, 2012b). But as indicated by the response pattern to question 34 most of the students did not realize that choice of origin is irrelevant for $\vec{\omega}$ of a rigid body rotating about a fixed axis. The corresponding distractor *b* was incorrectly chosen by 18% of students. Around 28 % of the students seemed to think that the choice of origin is significant for both the particle and the rigid body as encompassed by the incorrect option *c*.

Table 6.3: Boldfaced numbers indicate the correct choice. The questions are listed in Appendix A. DL denotes the difficulty level. The sample (S4) was described in chapter 2.

	S4 (N = 384)				
Q. No.	*a*	*b*	*c*	*d*	DL
26	108	160	**51**	57	.14
27	68	40	**191**	82	.50
28	**242**	29	26	77	.63
29	49	**270**	29	27	.72
30	46	44	56	**234**	.62
31	**196**	73	70	41	.52
32	48	**118**	84	130	.31
33	60	41	203	**71**	.19
34	**57**	82	179	61	.15
35	**203**	99	54	23	.54
36	148	**166**	22	41	.44
37	38	**229**	78	33	.61
38	18	147	**133**	77	.35

6.4 Administration and analysis - University of Washington sample

We administered the questions to 384 calculus based introductory level physics students at the University of Washington (UW). The questions were given as an online test after the lectures on rotational motion. They did not undergo tutorials on rotational motion (McDermott et al., 2002). The frequency with which distractors to each item were chosen is shown in table 6.3. It can be seen that the popular distractors to most items are similar to the Indian population. For example consider question 26. The most popular distractor is *b* followed by *a* for both American and Indian students. Such similarities can be seen for items 27, 28, 31, 33, 34, 35, 36, 37 and 38 as well. Consider item 38 as the second example. Choice *b* was the overwhelmingly popular distractor for the UW sample. Same was the case with the Indian students. These similarities hint that most of the difficulties identified in Indian students may also exist among American students.

However there exist some significant differences between the Indian and the

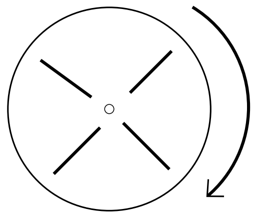

Figure 6.1: Schematic of a merry-go-round as viewed from above.

American students on questions 29, 30 and 32. The questions 29 and 30 investigate whether students can differentiate between linear and angular variables. The UW students seems to differentiate angular velocity from linear velocity as suggested by their high DL indices to these items. This may be because they have undergone tutorials in linear mechanics which address similar pitfalls (McDermott et al., 2002). Question 32 pertains to the direction of angular acceleration. Indian students tended to ascribe aspects of linear velocity to angular velocity. When it comes to the direction of the angular velocity, however a significant majority of the UW students consider it to be clockwise or counter-clockwise. This was revealed by the response to the following question, administered later to the same set of students.

1. Two students are pushing a merry-go-round so that it is spinning clockwise (when viewed from above, as depicted in figure 6.1) with uniform speed. Which of the following options describe the direction of the angular velocity of the merry-go-round ?

    (a) Vertically upward (out of the plane of the paper).

    (b) Vertically downward (into the plane of the paper).

    (c) Clockwise.

    (d) Counter-clockwise

Out of 378 students, 58 % chose *c* which states that the direction of the angular velocity of a merry-go-round is clockwise in contrast to the 27 % opting the correct choice *b*. Such a response from the students may result from a tendency of instruction (in high school and, even at the introductory level) to gloss over the vector nature of the angular velocity. There is a tendency for introductory physics instruction to describe rotation as being either clockwise or counter-clockwise, without reference to direction.

6.5 Concluding remarks

An inventory comprising 13 items on rotational kinematics of rigid body about a fixed axis was developed. The development of the inventory was based on insights from verbal data collected from the students, which helped in the construction of appropriate items and distractors. In the pilot studies we also sought brief explanations. These processes were described in chapter 2. The inventory after validation was administered to a set of teachers and two groups of students in India. The unsatisfactory performance of the teachers is alarming. Plausible causes include lack of meaningful teacher training programs, unimaginative evaluation systems and other socio economic issues (Mashood and Singh, 2012c). Moreover science education research has in general not yet percolated to the Indian educational system and thus has had limited impact. As described earlier the teachers were mainly from district and block level schools or colleges. The above mentioned problems are more acute in their case as compared to urban teachers.

The response patterns of the students were analyzed in detail. We identified difficulties and pitfalls in reasoning. Aided by interviews with students they were categorized into four broad themes. These include inappropriate extensions of familiar procedural practices, reasoning cued by primitive elements in thought, lack of differentiation between related but distinct concepts and indiscriminate use of equations. The difficulty level was calculated for each item. Response pattern of the introductory level students at UW exhibit similarities with that of Indian students. Popular distractors to most of the items were same. This suggests that further research into the cross-cultural characteristics of the difficulties represented by these distractors would be fruitful.

Chapter 7

Evaluating the inventory on rotational kinematics

7.1 Introduction

Inventory construction is a systematic and iterative process. The general protocols and methodologies involved in the development of the inventory were discussed in chapter 2. The content evolution of items constituting the three parts of the inventory were described in the subsequent chapters 3, 5 and 6. The first phase of administration discussed in these chapters yielded interesting results. The administration was followed by interactions with a subset of students. The insights obtained from the interactions helped in the analysis of the response patterns of students. They also fed into refinement of the inventory. Some of the items and distractors underwent minor modifications. A new item was added which brought the total number of items to 39.

It is important to administer the test to larger and varying samples. This helps in the evaluation of the inventory as a diagnostic and assessment tool. Keeping this in mind we administered the inventory to over nine hundred higher secondary school students from different parts of India. The sample S3 (N=905) was described in chapter 2. This constituted the second phase of administration of the inventory. Like the first phase, interviews with a subset of students (N=35) were carried out. The aims included verification of the inferences we made by analyzing the response patterns of students in phase 1, which were discussed in chapters 3, 4 and 6. They also served as further validation interviews for the items. It may be recalled that in addition to face validation by students, the inventory was content validated by experts.

The average performance and the difficulty level of each item for the first phase of administration have been tabulated in chapters 3, 5 and 6. However the large sample size (S3, N=905) enables us to carry out a detailed analysis. This will help us to gauge the strengths and weaknesses of the inventory, both locally (item wise) and globally (whole test). It is also important that reliability of the inventory needs to be established. The administration of part of the inventory to students at the University of Washington revealed the consistency of the response patterns with varying samples (see chapter 6). This reproducibility of the results for the third part of the inventory indicates reliability. We considered it important to investigate reliability of the whole test using commonly employed statistical indices like the Kuder Richardson reliability index (Ding et al., 2006; Ding and Beichner, 2009). In addition we decided to perform other item-wise and whole test statistical analyses. These analyses constitute the theme of this chapter.

Section 7.2 briefly describes the sample and the inventory. It is followed by a discussion of the difficulty level, index of discrimination, point-biserial coefficient, Ferguson's delta and Kuder-Richardson reliability index for the inventory (Ding et al., 2006; Ding and Beichner, 2009). The difficulty level, index of discrimination, point-biserial coefficient are calculted for each item. We have discussed difficulty level earlier. The index of discrimination and point-biserial coefficient measures the extend to which an item can discriminate between low and high scoring students. In contrast Ferguson's delta measures the discriminatory power of the whole test.

Item response curve (IRC) analysis is described in section 7.3. Item response curves are visually rich and versatile (Morris et al., 2006). They provide information about items and samples which the statistical indices mentioned above does not. Sections 7.2 and 7.3 also include a detailed discussion of a few representative items along with insights obtained from the interviews with the students. Section 7.4 constitutes a brief conclusion.

7.2 Statistical analyses

The sample for the first phase (discussed in chapters 3, 4 and 6) of administration was mainly from schools in the Mumbai region. For the second phase we decided to carry out a pan Indian administration involving a large number of students. The inventory was administered to a total of 905 students from Jaipur, Patna, Mumbai, Hyderabad and Bangalore. Details of this sample S3 are described in chapter 2. The inventory was administered in English. All the students had been taught rotational motion. There was no strict time restriction imposed on the students.

The inventory had 39 items. It may be recalled that the items broadly fell under three categories namely rotational kinematics of a particle (19 questions), rotational kinematics of a particle in rectilinear motion (7 questions) and of a rigid body rotating about a fixed axis (13 questions). The items are listed in appendix A. There were 4 choices to each item with only one correct answer. Students were asked to answer all items by selecting the most appropriate choice to each. Some of them left a few items unanswered despite the request. Evaluation was done by assigning one mark to the correct choice to each item. There was no negative marking for the wrong choice. Administration was followed by semi-structured interviews with a group of students. In total around 35 students participated in the interviews. They were done individually or in small groups of 2-3 students. The items were validated once again by the student interviews. It ensured that the items and distractors were interpreted as intended. Some insights from the interviews are discussed in this section and the next.

The average score of the students was 18.4 (47.18 %). The standard deviation σ_x = 10.1 (see table 7.1). This is high and may be attributed to the broad distribution of the total scores among the student population. This distribution of scores is depicted in figure 7.1. The graph has a dominant peak at the lower end centered around the score of 10. It is followed by a relatively flat region in the score range 16 to 22. Note that the average lies in this flat region. There are intermittent peaks to the right. The distribution indicates a broad underlying bimodality. There were 35 students with the median score of 15 with 441 students below and 429 above it. The number of students with a total score of 4 or below was 15 (1.7 %). The number of students with a total score of 35 or above was 81 (9 %) and is large. The peak above the score of 35 in figure 7.1 attests to this.

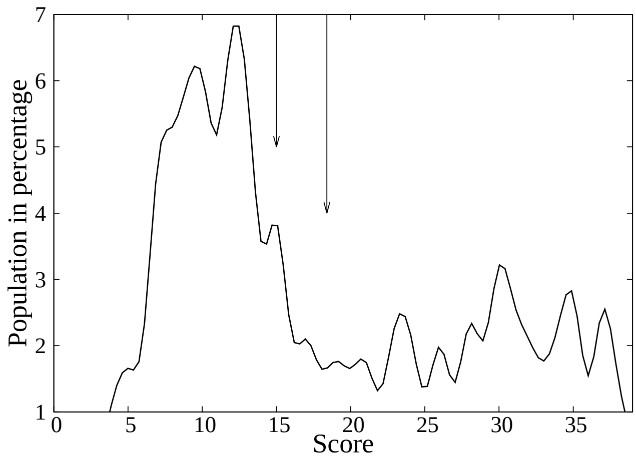

Figure 7.1: Distribution of scores of students (N=905) in the inventory. The short and the long arrows indicate the median (15) and the average (18.4) respectively.

The other whole test statistics we calculated include Ferguson's delta and Kuder Richardson reliability index which are given in table 7.1. Ferguson's delta (δ) is given by

$$\delta = \frac{N^2 - \Sigma {f_i}^2}{N^2 - N^2/(K+1)}. \tag{7.1}$$

Here N denotes the number of students in the sample, f_i is the frequency of each of the total score (0 to 39 in our case) and K is the total number of items in the inventory. The index is a measure of the discriminatory power of the whole test. Kuder Richardson reliability index (r_{test}) is given by

$$r_{test} = \frac{K}{K-1}(1 - \frac{\Sigma P_i(1-P_i)}{{\sigma_x}^2}). \tag{7.2}$$

Here K is the total number of items in the inventory, P_i is the difficulty level (DL) of the item i and σ_x is the standard deviation of the total score. The index measures the self-consistency of the test. A detailed description of these indices and their theoretical underpinnings can be found elsewhere (Ding et al., 2006, Ding and Beichner, 2009). The value of δ for the test is 0.99 while the desired value should be ≥ 0.9. The r_{test} index was obtained to be 0.93 which also is well above the desired value of ≥ 0.8. Thus the test did well on these indices. Note that the average was around 50 % which is optimal.

Table 7.1: The average score, associated standard deviation, median, Ferguson's delta and Kuder Richardson reliability index for the whole inventory. There were 39 items. Evaluation was done by giving 1 mark to the correct answer and there was no penalty for wrong choices. The total number of students in this sample (S3) was N=905.

Test statistics	value
Average score	18.40 (47.18 %)
Standard deviation	10.01
Median	15
Ferguson's delta	0.99 (desired value, ≥ 0.9)
Kuder Richardson reliability index	0.93 (desired value, ≥ 0.8)

The detailed response pattern of the students is given in tables 7.2 and 7.3. It may be noted that the response pattern to items 26-38 was also given in chapter 6. It is repeated for completeness and for an extended discussion in this chapter along with the rest of the inventory. The frequency of choice of distractors to all items is provided. In the first phase of administration we made our inferences partly based on the frequency with which particular distractors were chosen. For example, to question 6 the distractor *a* was crafted to incorporate an instance of indiscriminate use of equations which we observed during the developmental phase. A significant number of students selected this choice which indicates that this is indeed a recurring pattern. It may be noted that all items were validated by student interviews. Broadly the response pattern of students depicted in tables 7.2 and 7.3 is consistent with those observed during phase one. However the percentage of students getting the answers right has gone up for most of the questions. This can be inferred directly by comparing the values of difficulty level for phase 1 and phase 2. Difficulty level (DL or P) is defined as the ratio of the number of correct respons to the total number of students who attempted the question (Ding et al., 2006; Ding and Beichner, 2009). Note that the term is a misnomer and the value actually indicates the easiness level. The index can also be construed as the average score on the item. Observation of tables 7.2 and 7.3 makes clear the increase in P value for all items in phase 2. For example to question 1 the difficulty level increased to 0.64 compared to 0.53 in phase 1. The difficulty level of question 2 increased similarly to

0.38 from 0.27. The value of P for all items except 22 and 39 are above the desired value of 0.3. For these two items the P indices are 0.25 and 0.26 indicating that they are of relatively high difficulty. The difficulty level averaged over all items is 0.48.

Apart from the difficulty level (P) we also calculated the discrimination index (D) and the point biserial coefficient (r_{pbs}) for each item (see tables 7.2 and 7.3) The item discrimination index is given by

$$D = \frac{N_H - N_L}{N_{27}}. \tag{7.3}$$

Here N_H denotes the number of correct responses to the item from the top 27 %, N_L the number of correct responses to the item from the bottom 27 % and N_{27} is the number of students constituting 27 % of the sample. In our case $N_{27} = 27N/100 = 244$ (N=905). The index D measures the extend to which an item can discriminate between low and high scoring students (Ding et al., 2006; Ding and Beichner, 2009). The desired range for D is 0.3 to 1. For all items except 6 the values are above 0.3 as can be seen from tables 7.2 and 7.3. The average value of D for all items is 0.65. The point biserial coefficient of an item is given by

$$r_{pbs} = \frac{\bar{X}_1 - \bar{X}_0}{\sigma_X}\sqrt{P(1-P)} \tag{7.4}$$

where $\bar{X}_1$ is the average of the total scores of those students who have correctly answered the item, $\bar{X}_0$ is the average total score of all the students who have incorrectly answered the item, σ_X is the standard deviation of the total scores and P is the difficulty level of the item. The index is a measure of correlation of students' score on the item and score in the test (Ding et al., 2006; Ding and Beichner, 2009). High r_{pbs} for an item implies that students whose total scores are high are more likely to get it correct. The desired value for r_{pbs} is ≥ 0.2. As can be seen from tables 7.2 and 7.3 the value of r_{pbs} for all the items except 6 are above the desired value. The average value of r_{pbs} is 0.53. Item 6 will be subjected to further analysis based on the IRCs in the next section.

Let us make a few observations about the pattern of frequency of distractors given in tables 7.2 and 7.3. The items are listed in the appendix A. As mentioned

Table 7.2: Boldfaced numbers indicate the correct choice. The sample size is N=905 for S3. Some candidates left a few questions unanswered despite being requested otherwise. The desired values for difficulty level (DL or P), item discrimination index (D) and point biserial coefficient (r_{pbs}) are ≥ 0.3, ≥ 0.3 and ≥ 0.2 respectively. The difficulty level (DL or P) in parenthesis is the value from phase one of administration for S1 which had N=79.

Q. No.	*a*	*b*	*c*	*d*	DL or P	D	r_{pbs}
1	153	108	**579**	65	0.64 (0.53)	0.65	0.50
2	55	239	264	**342**	0.38 (0.27)	0.61	0.48
3	**503**	104	143	142	0.56 (0.44)	0.58	0.42
4	**412**	159	197	134	0.46 (0.37)	0.79	0.63
5	96	98	259	**446**	0.50 (0.28)	0.88	0.72
6	312	192	**279**	102	0.31 (0.41)	0.19	0.12
7	**585**	99	69	151	0.65 (0.52)	0.52	0.42
8	45	124	**598**	142	0.66	0.65	0.51
9	80	220	**377**	223	0.42 (0.32)	0.70	0.55
10	95	43	**692**	73	0.77 (0.56)	0.53	0.45
11	**624**	82	147	47	0.69 (0.58)	0.64	0.52
12	62	155	**441**	246	0.49 (0.37)	0.78	0.64
13	168	**487**	185	57	0.54 (0.41)	0.70	0.56
14	194	98	96	**510**	0.57 (0.43)	0.75	0.59
15	91	**497**	187	126	0.55 (0.37)	0.71	0.54
16	99	224	164	**408**	0.46 (0.35)	0.70	0.57
17	133	89	237	**430**	0.48 (0.15)	0.82	0.65
18	323	95	149	**326**	0.37 (0.13)	0.78	0.65

earlier they are broadly consistent with the response of group S1 in the first phase. For instance consider item 1. The item probes student understanding of the angular speeds of two points on the seconds hand and a point on the minute hand of a clock. The popular distractors to the item were *a* and *b* in phase 1. Distractor *a* incorporates the notion that angular speeds are different for all points like linear speeds. While distractor *b* states that angular speeds are equal for all points similar to that on a rigid body rotating about a fixed axis. The distractors *a* and *b* remained the most popular choices in phase 2 as well. The percentage of population making the choices however varied compared to the earlier phase. For this item 20.3 % chose *a* in phase 1 while it decreased to 16.9 % in phase 2. Similarly choice *b*

Table 7.3: Boldfaced numbers indicate the correct choice. The sample (S3) size N=905. Some candidates left a few questions unanswered despite being requested otherwise. The desired values for difficulty level (DL or P), item discrimination index (D) and point biserial coefficient (r_{pbs}) are ≥ 0.3, ≥ 0.3 and ≥ 0.2 respectively. The difficulty level (DL or P) in parenthesis is the value from phase one of administration for S1 which had N=79.

Q. No.	*a*	*b*	*c*	*d*	DL or P	D	r_{pbs}
19	221	**320**	216	143	0.36 (0.14)	0.40	0.32
20	141	117	**522**	122	0.58 (0.34)	0.81	0.64
21	366	101	**312**	124	0.34 (0.22)	0.62	0.51
22	306	212	146	**226**	0.25 (0.16)	0.40	0.38
23	**301**	251	247	100	0.33 (0.13)	0.42	0.36
24	244	209	**274**	162	0.31 (0.14)	0.54	0.43
25	**326**	151	245	164	0.37 (0.23)	0.64	0.49
26	159	350	**271**	101	0.31	0.34	0.27
27	171	106	**457**	151	0.52	0.84	0.70
28	**526**	78	101	186	0.59	0.74	0.61
29	90	**529**	146	136	0.59	0.78	0.64
30	206	151	104	**436**	0.48	0.85	0.70
31	**507**	145	168	74	0.57	0.77	0.61
32	171	**439**	182	98	0.49	0.81	0.66
33	137	114	320	**316**	0.36	0.67	0.57
34	**333**	164	255	127	0.38	0.46	0.39
35	**567**	110	110	114	0.63	0.69	0.53
36	242	**423**	94	135	0.47	0.76	0.63
37	90	**504**	240	69	0.56	0.72	0.57
38	95	352	**374**	67	0.42	0.57	0.48
39	270	248	133	**231**	0.26 (0.11)	0.50	0.50

registered a decrease from 17.7 % to 11.9 %. This decrease can be attributed partly to the increase in percentage of the correct choice in second phase. Because of the large and varied sample size phase 2 the inferences based on phase 2 have a greater authenticity. Some of the candidates who chose *a* and *b* were among those who were interviewed. About six students who chose *a* displayed a poor understanding and high level of confusion with regard to angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$). They were unable to articulate the difference between linear and angular velocity. Around 8 interviewed students who chose *b* showed some understanding about $\vec{\omega}$ and $\vec{\alpha}$. They knew that these quantities were distinct from linear variables and had vague notions about the concepts. They said they chose *b* because they had learnt that all particles on the rigid body have same $\vec{\omega}$ and the hands of the wall clock are rigid bodies.

As another illustration consider item 14. The question probes the angular acceleration of a pendulum bob at an extreme position when its velocity is zero. The most popular distractor was *a* in phase 1. It states that the angular acceleration at the point is zero. This pitfall is similar to one observed in linear kinematics where students erroneously think that the linear acceleration of the bob at the extreme position is zero since its linear velocity is zero. In phase 2 *a* remained the most popular distractor indicating the consistency of the pattern. The percentage of students selecting the choice however dropped from 31.6 to 21.4 in phase 2. This aspect was verified in the interviews. Those who opted *a* to the item were asked about the linear acceleration at the point of maximum height of a ball thrown vertically upwards. They all said that linear acceleration at that point was zero.

For a few items though the popular distractors remained same but the order of their preference changed. Item 2 is an example. The item probes students' ability to visualize the motion (clockwise - anticlockwise) of the second hand of a transparent clock from the front and rear side and also to ascertain the direction of angular velocity of the tip of the clock hand. The popular choices in phase 1 were *b* (42 %) and *c* (25 %). Distractor *b* states that the seconds hand moves clockwise when viewed from the front side and the direction of angular velocity of its tip is out of the plane of the clock. On the other hand distractor *c* states that the seconds hand moves anticlockwise when viewed from the rear side. The second part of the distractor is identical to that of *b*. This serves to elicit the incorrect notion that the direction of angular velocity of the tip of clock hand changes with the orientation of motion (clockwise - anticlockwise) depending on whether we view the clock from the front or the rear. Unlike phase 1, in phase 2 choice *c* (29 %) was slightly more popular than *b* (26 %).

Another interesting observation from tables 7.2 and 7.3 is that the second phase of administration ferreted out certain distractors chosen by an overwhelming majority. Example includes distractor *a* to item 21. This choice was selected by 366 students (40.4 %) which is greater than the percentage of students who correctly answered the item (34 %). The distractor corresponds to the incorrect notion that angular acceleration of a particle moving in a straight line is zero (origin not on the line of motion). A second example is option *c* to item 33 which was chosen by 320 candidates (35.4 %). The distractor corresponds to a popular misconception that an angular acceleration always implies a torque. The large sample size implies that such a notion is indeed widespread. The overwhelming appeal of these distractors were mirrored in the interviews as well. There are cases where all the three distractors were chosen in nearly equal proportions. Item 27 serves as an illustration. The item probes student understanding of the operational definition of the angular velocity of a rotating pulley. Distractors *a*, *b* and *d* were chosen by 171 (18.9 %), 106 (11.7 %) and 151 (16.7 %) candidates respectively. Distractor *a* states that the angular velocity is the vector sum of angular velocities of all the particles constituting the body. Choice *b* similarly states that it is the scalar sum of the magnitude of angular velocities of all the particles. Distractor *d* maintains that angular velocity of the pulley is equal to the angular velocity of the center of mass of the pulley. Some other examples are items 3, 20, 29 and 35.

7.3 IRC analysis

An item response curve (IRC) is a visually rich versatile tool for analyzing student responses. IRC involves a plot of the percentage of students $P_i(\theta)$ selecting a choice i to an item vis-a-vis their ability θ. In our case there are four choices and i is *a, b, c* or *d*. We considered the total score of the students in the test to represent their ability level θ. A detailed description of the technique, its merits and theoretical underpinnings can be found elsewhere (Morris et al., 2006; Ding et al., 2006; Ding and Beichner,2009; Singh, 2009). The IRCs to all 39 items in our inventory were plotted (see figures 7.2 to 7.40). These items are listed in appendix A. We now discuss a few items in detail to illustrate how we can gainfully learn from IRCs. To begin with consider item 2 which was discussed in the previous section. Figure 7.3 depicts the IRCs plotted for all choices to the item. Fluctuations apart, the correct choice d to the item correlates positively with the ability level as can be seen. To facilitate a clear interpretation we modeled it by the logistic response function given by

$$P_d(\theta) = s + \frac{(100 - s)}{1 + exp[-(\theta - m)/w]}. \tag{7.5}$$

Here s indicates the percentage of students with low ability who will respond correctly to the item. Note that for $\theta << m$, $P_d(\theta) \rightarrow s$. The parameter m refers to the ability level corresponding to the inflection point $P_d(\theta = m) = 50 + s/2$. Students with ability level higher than m are more likely to pick the correct choice. The factor *1/w* gives the slope of the curve at m. Smaller the w steeper is the slope and the curve approaches a step function. A steeper slope means that the item sharply segregates low ability ($\theta < m$) students from high ability ($\theta > m$). The fit to the correct choice d to item 2 is a sigmoid (m=28.41, s=21.53, *1/w* =.25) which remains flat till the ability level $\theta = 18$. The percentage of students opting d steadily increases after that. The slope starts decreasing after a score of 34. The choice a is insignificant as can be inferred from the low lying curve close to the x-axis. A closer look at the item response curves to the distractors reveals that b and c constitute the prominent wrong choices. However IRCs help us to meaningfully distinguish between these two distractors. IRC of distractor b is prominent in the ability range [3:15] while that of c is more popular among ability levels greater than 17. Thus the distractors b and c exhibit discriminatory power. Analysis of the content of these distractors reveals that choice c is proximate to the correct answer than b. Choice c demands visualization of movement of the hand of a transparent clock from the rear side and also ascertain the direction of its angular velocity. On the other hand b requires the same analysis by looking at the clock directly. Item 28 constitutes another instance where aspects of visualization are probed.

As illustrated above the discussion of IRCs give perspectives on student difficulties not provided by tables 7.2 and 7.3. It also allows us to gauge the quality of the items. Consider item 5 as the second example. The item probes the direction of angular velocity of a planet at a point on its elliptical trajectory (see appendix A). The indices for the item are good with optimal difficulty level of 0.5 and high values of D and r_{pbs} indicating discrimination. This is mirrored in the IRC for the correct choice *d* which is an excellent sigmoid (*m*=17.6, *s*=8 and *1/w*=.24) as can be seen from figure 7.6. The phase 1 analysis revealed that all three incorrect options *a*, *b* and *c* are chosen by a significant number of students. Effectively all these options convey that the direction of angular velocity is in the plane of motion, with varying orientation. Data from the second phase of administration in table 7.2 however indicates an overwhelming appeal for distractor *c*. This aspect is made visually vivid by the peak characterizing choice *c* in the lower ability range from 2 to 16. The curves for *a* and *b* are overlapping and lie low across the full range from score 3 to 33. It may be noted that the distractor *c* states that the direction of angular velocity at the point on the trajectory is tangential to the point. All three distractors together would indicate fixation with the inappropriate prototype that direction of angular velocity should be in the plane of motion. Distractor *c* alone is more specific in that it brings out the case where angular velocity ($\vec{\omega}$) direction is confused with the direction of linear velocity ($\vec{v}$). As such this is also an instance of lack of differentiation between related but distinct concepts ($\vec{\omega}$ and $\vec{v}$). Other items which reveal this pattern include items 29, 30, 31 and 32.

Item 6 is another example which is posed in the same context as item 5 (see appendix A). The item probes student understanding of the equation $\vec{v} = \vec{\omega} \times \vec{r}$ and its validity conditions (here $\vec{r}$ is the position vector). While discussing the indices in table 7.2 earlier we mentioned that the discrimination index D and point biserial coefficient r_{test} to this item are below the desired values. This is because of the high difficulty of the item as evident from the difficulty level (P=0.31). The IRCs given by figure 7.7 clarifies this difficulty aspect distinctly. Figure 7.7 clearly shows that only those above the score 37 consistently gave the correct response to this item. The IRC for the correct answer is a sigmoid (*m*=37.4, *s*=33.5 and *1/w*=2.8) with a long flat tail and a high slope near the right extreme. This abnormal shape and the low indices are not in anyway due to errors in the content matter of the item. One can see that *a* and *b* are the widely chosen distractors. As per distractor *a* the equation holds for a planet moving in an elliptical orbit and follows from the definition of angular velocity. While distractor *b* simply states that the equation is true because the planet is in rotational motion. The IRCs however reveal that the population choosing them are relatively distinct. Choice *a* has more appeal to higher ability students as evident from the imposing peak of its IRC in the range [27:37]. The curve to choice *b* has a relatively smaller peak in the range [6:17]. The choices are indicative of lack of knowledge of validity condition associated with the equation under consideration. The equation is valid only for circular motion. During interviews we found that most of the students had learnt the equation by rote without understanding its derivation. The basis of an equation and its validity is often made clear by understanding its derivation. The interviews revealed that students who were incorrect about the equation $\vec{v} = \vec{\omega} \times \vec{r}$ also got item 39 wrong. Item 39 probed student understanding of the relation $\vec{a} = \vec{\alpha} \times \vec{r}$. Items 19, 23 and 36 also reveal similar patterns of indiscriminate use of equations.

Item 14 was discussed in the previous section as revealing an instance of a pitfall paralleling those found earlier in linear kinematics. The incorrect notion was that acceleration of the pendulum bob is zero at the extreme position when its velocity is zero. The IRC of the corresponding distractor *a* being significant in the region [5:17] is a visual confirmation of this (see figure 7.15). In addition to the visual display, it may once again be noted that IRC provides the ability range of students harboring this notion. This information is not provided by table 7.2. The other distractors *b* and *c* are relatively insignificant as revealed by their IRCs. The curves are overlapping, almost flat and close to the x-axis throughout the range from 2 to 35. Distractor *b* states that the angular acceleration at a single instant is undefined while *c* states that the angular acceleration at a single position is undefined. The curve corresponding to the correct choice *d* follows a sigmoid (m=19, s=20 and $1/w$=.22). The sigmoid is flat till the score 10 and then steadily rises till 30. The slope of the curve is indicative of its relatively good discriminatory power. After score 30 the slope decreases and the curve once again turns flat. Items 3, 7-11 and 15-18 reveal some other instances of pitfalls paralleling those found earlier in linear kinematics.

Item 19 constitutes another interesting example. The item probes student understanding of the angular speed of a particle moving in a straight line with constant velocity. The origin is not on the line of motion. The item is relatively difficult with a P value of 0.36. The IRC to the correct choice *b* in figure 7.20 displays this aspect clearly. Choice *b* maintains that the angular speed decreases as one moves away from the origin because of a corresponding decrease in the angular displacement. The fit to *b* is a sigmoid (*m*=33, *s*=24 and *1/w*=.94) and has a long tail parallel to x-axis till the score 30. The IRC then rises sharply in the range 30 to 35 and remains flat further ahead. The high difficulty of the item is thus displayed vividly by the steep slope of the curve in the range 30 to 35. One can further see from figure 7.20 that IRCs to all the distractors have peaks significantly above the tail of the sigmoid. Clearly all the three incorrect choices *a*, *c* and *d* represent important pitfalls in understanding among students. Distractor *a* correctly states that the angular speed is decreasing but bases the argument on the equation $v = \omega r$ which is not valid in the context. IRC to *a* has its peak from score 15 to 30. Distractor *c* states that the angular speeds at two points on the line of motion are equal since the particle has no linear acceleration. Students erroneously think that zero linear acceleration implies zero angular acceleration for this situation. The IRC to *c* almost overlaps with that of *a*. However the peak of *c* is higher and extends till the score of 34. Choice *d* states that the angular speed of the particle is zero because the motion is linear. The choice appealed mainly to lower ability students in the range 2 to 18. The reluctance to ascribe angular quantities to a particle in linear motion is indicative of fixation to the prototype that rotational motion is necessarily circular. In our interviews we found that many students were emphatic against associating angular quantities with a particle in linear motion. Some persisted in this notion even after we gave hints that they may be wrong. They said that linear and circular motion are two distinct kinds of motion. Some other patterns of fixation with inappropriate prototypes are revealed by items 4, 5, 12, 13 and 21.

Item 26 investigates student understanding of the operational definition of the magnitude of angular velocity of a rigid body rotating about a fixed axis. In other words how the angle $\Delta\theta$ in the relation $\omega = \Delta\theta/\Delta t$ is identified. The answer to the item is given in the choice *c* which states that the angle $\Delta\theta$ is traced by a line perpendicular to the axis from any particle on the body. IRC to *c* given by figure 7.27 however makes it clear that the question was difficult for a majority of the students. This is indicated by the long tail to the fit (*m*=34, *s*=20 and *1/w*=.4) till the score of 28 and the subsequent rise thereafter. The most popular distractor is b. IRC for this is a curve spanning the entire range. The peak lies between scores 15 and 35. The choice states that the angle $\Delta\theta$ is traced by the position vector of any particle on the body from a specified origin. It may be recalled that this is how angles are often traced. As such the choice b represents a case of a familiar procedural practice extended to a context where they are inappropriate (another instance of this pattern is illustrated by item 27). The other distractors a and d are chosen mainly by students in the lower ability range. Distractor a states that the angle $\Delta\theta$ is the angle traced by the position vector of the center of mass of the rigid body. While according to choice d, $\Delta\theta$ is traced by a line perpendicular to the axis from the center of mass only. Both choices signify the role of center of mass as a representative point for the rigid body. We found during our interactions with them that they are cued by the idea of center of mass being a representative point which resonates with certain primitive notions in their thinking. Instances of the same pattern are illustrated by items 27 and 28 as well.

One feature about the IRCs which we wish to draw attention are fluctuations. For the correct response, usually given by a sigmoid, one notices that the fluctuations are often prominent in the mid-ability range. For example consider IRCs to the items 13 and 14 which are of similar difficulty levels. In figure 7.14 of item 13 one can see a dip in the IRC to the correct answer b at the ability level $\theta = 24$. However in item 14 the IRC to the correct choice registers a peak at the same ability level ($\theta = 24$). Items 33 and 34 constitute another example. These items are also of similar difficulty levels. In figure 7.34 of item 33 one can see a peak in the IRC to the correct answer d at the ability level $\theta = 24$. However in item 34 the IRC to the correct choice registers a dip at the same ability level. Consider items 20 and 35 as a third example. Item 35 has a dip corresponding to the ability level 22 for IRC of the correct choice a. While at the same ability level IRC of the correct choice c to item 20 has a peak. These illustrations imply that students in the same ability may get one item right and the next one wrong. This vacillation perhaps is indicative of the confused thinking characteristic of students who are in a transition from novice state towards expertise, a point made by Bao and Redish (2006) using different arguments. It may be noted that the random noise decreases as one approaches higher ability levels. It can also be seen that in the flat region above the ability level

30 the fluctuations are often minimal (eg. items 1, 4, 5, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 27, 28, 29, 30, 31, 32, 35, 36, 37).

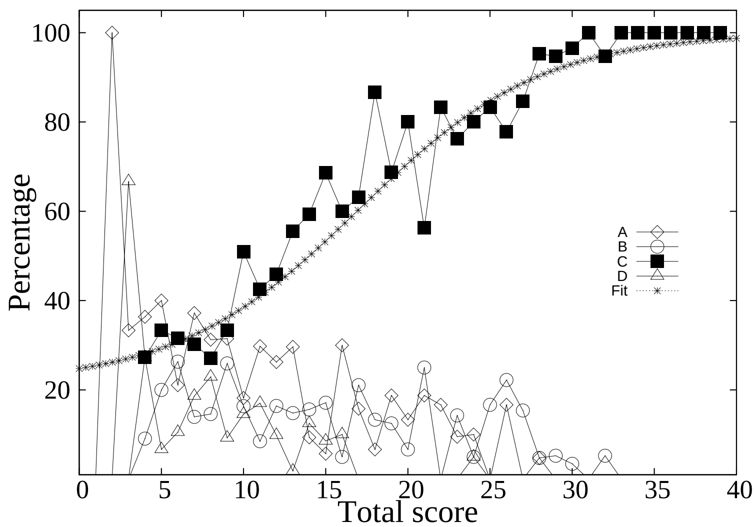

Figure 7.2: Item 1 - Item Response Curves

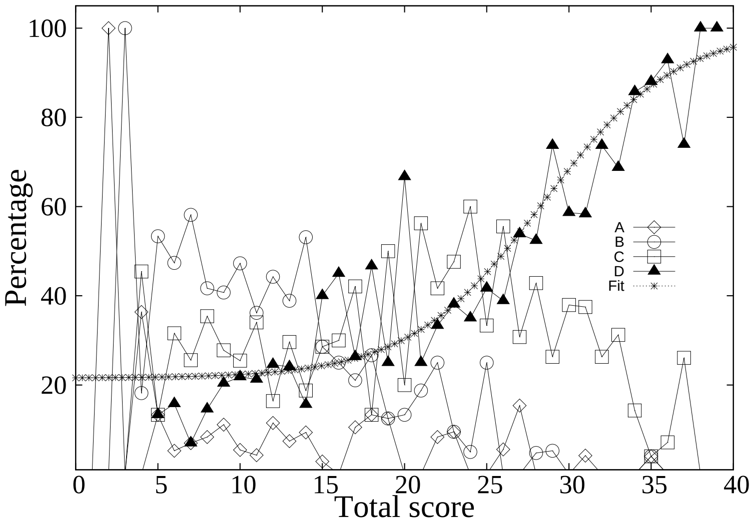

Figure 7.3: Item 2 - Item Response Curves

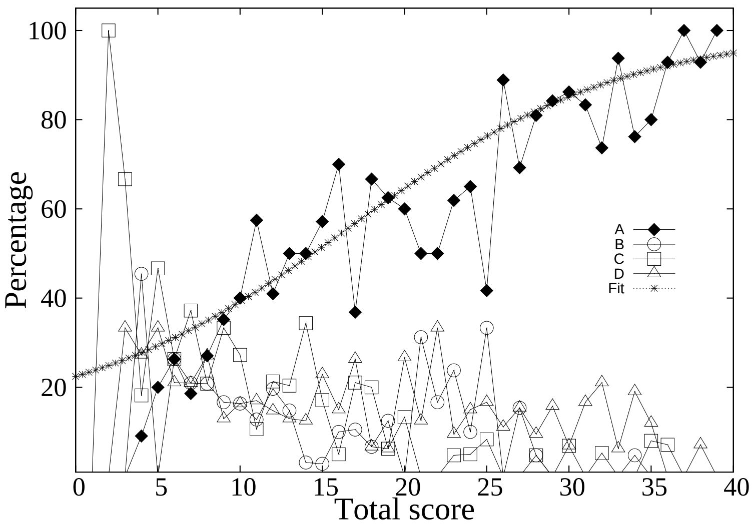

Figure 7.4: Item 3 - Item Response Curves

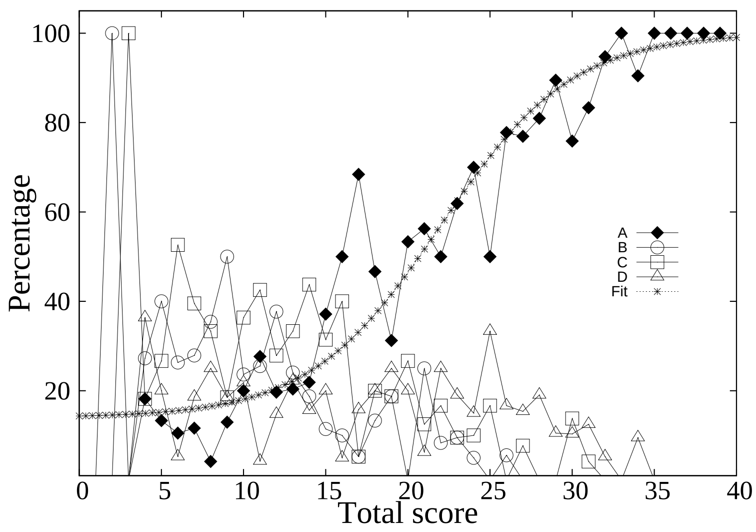

Figure 7.5: Item 4 - Item Response Curves

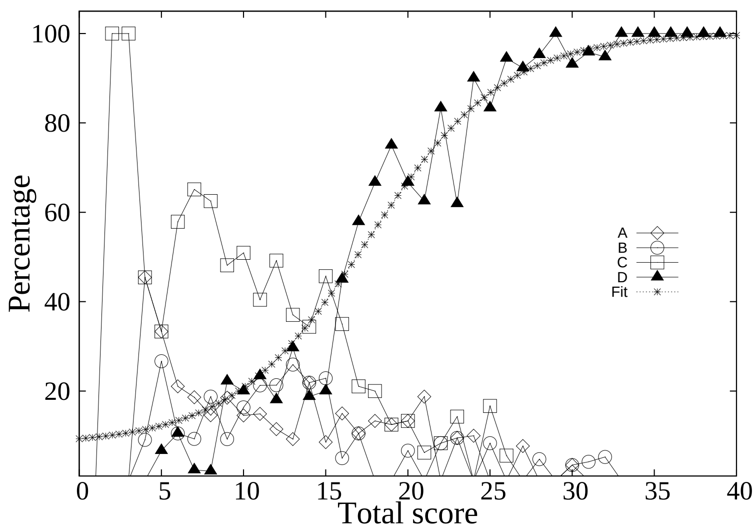

Figure 7.6: Item 5 - Item Response Curves

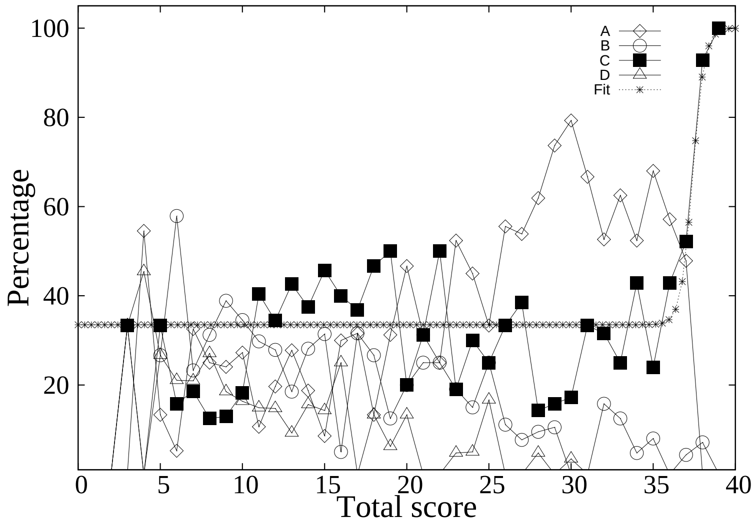

Figure 7.7: Item 6 - Item Response Curves

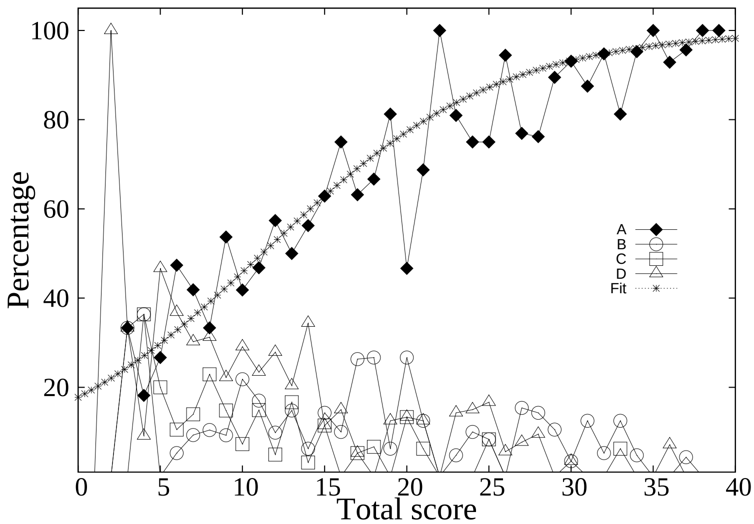

Figure 7.8: Item 7 - Item Response Curves

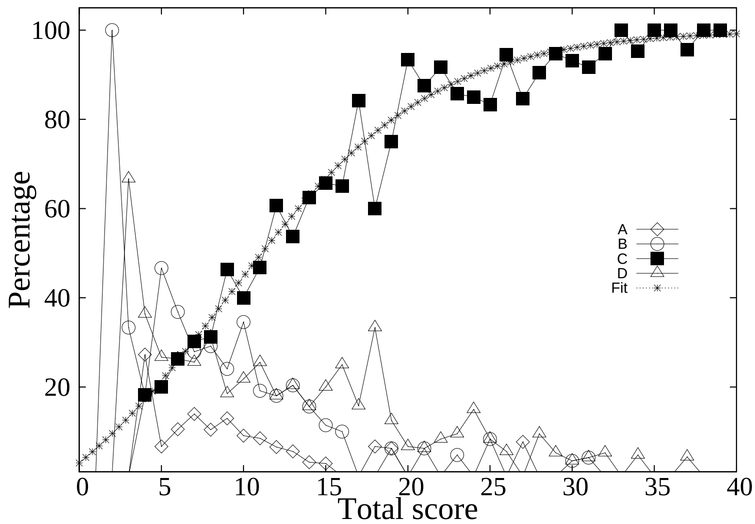

Figure 7.9: Item 8 - Item Response Curves

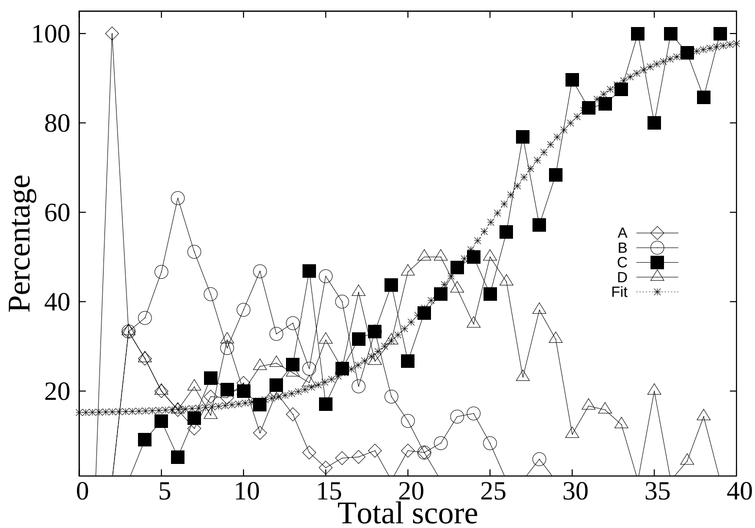

Figure 7.10: Item 9 - Item Response Curves

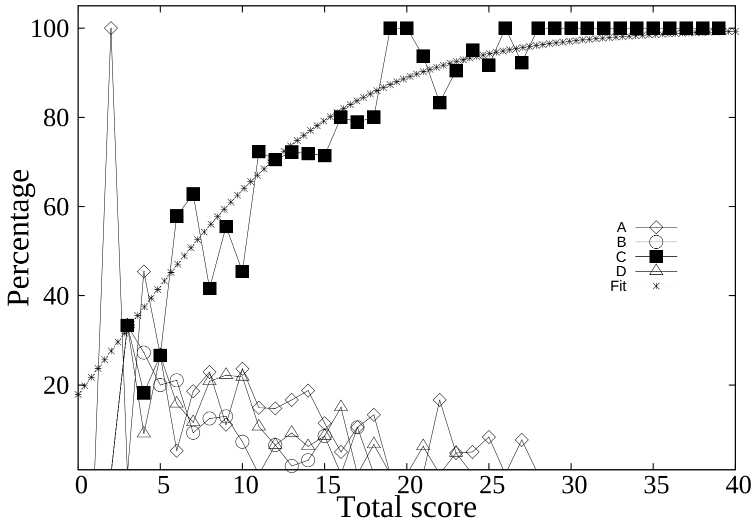

Figure 7.11: Item 10 - Item Response Curves

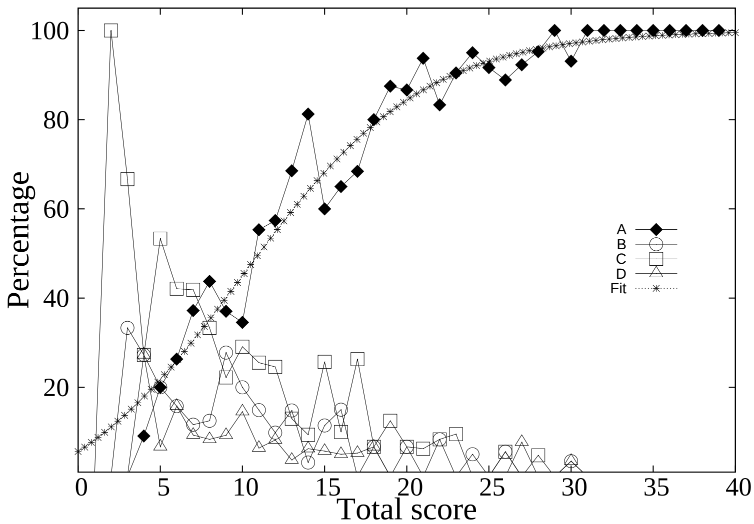

Figure 7.12: Item 11 - Item Response Curves

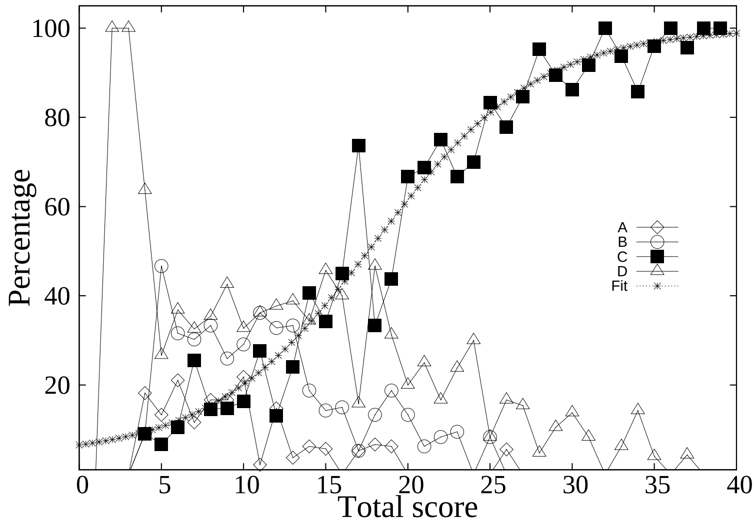

Figure 7.13: Item 12 - Item Response Curves

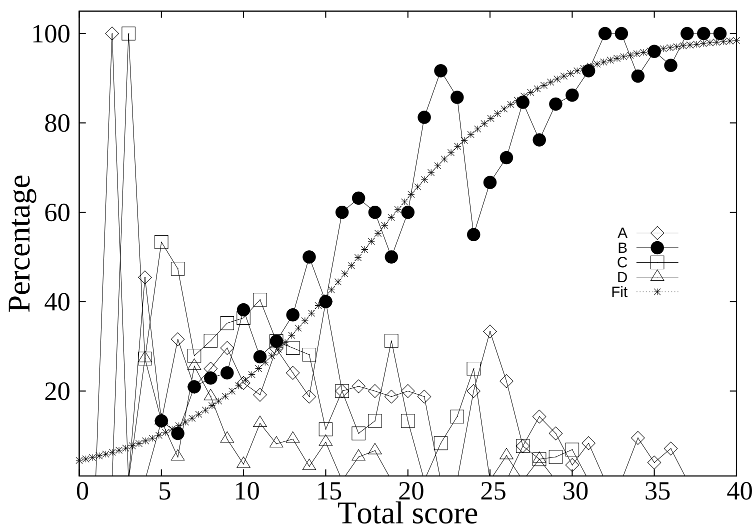

Figure 7.14: Item 13 - Item Response Curves

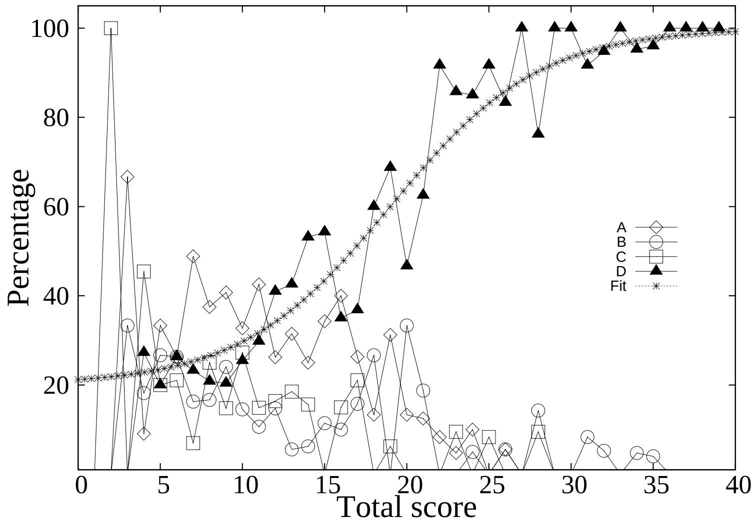

Figure 7.15: Item 14 - Item Response Curves

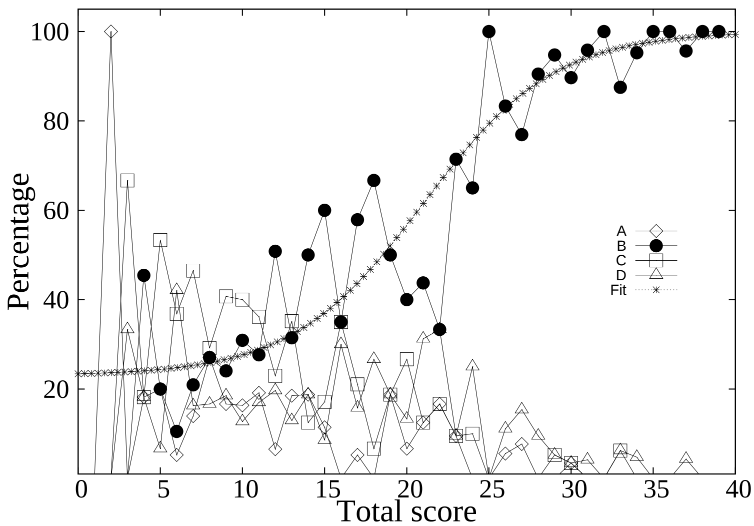

Figure 7.16: Item 15 - Item Response Curves

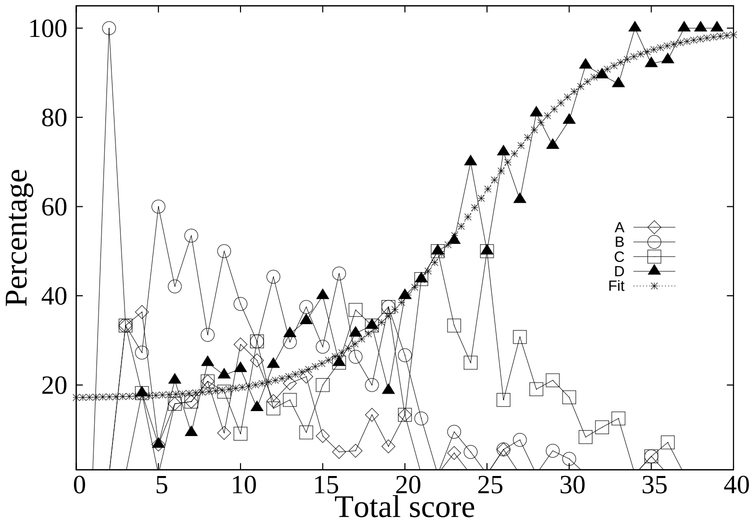

Figure 7.17: Item 16 - Item Response Curves

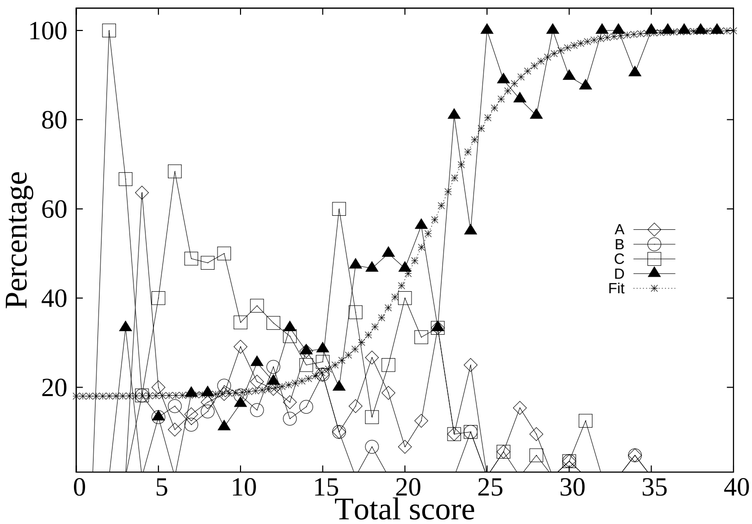

Figure 7.18: Item 17 - Item Response Curves

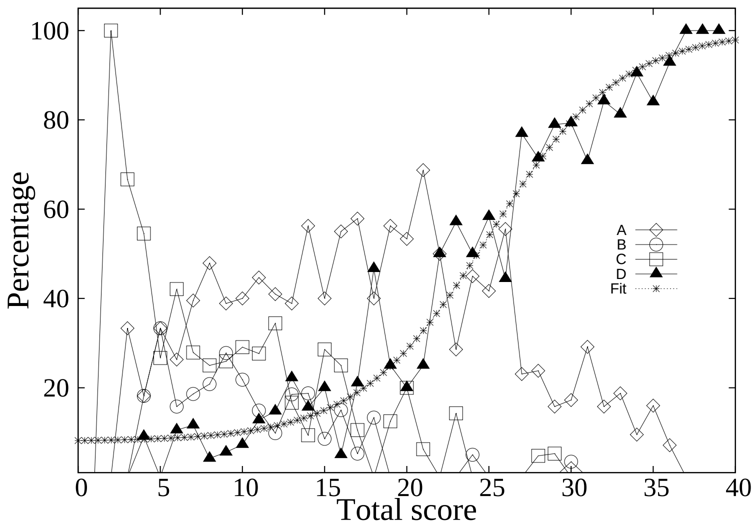

Figure 7.19: Item 18 - Item Response Curves

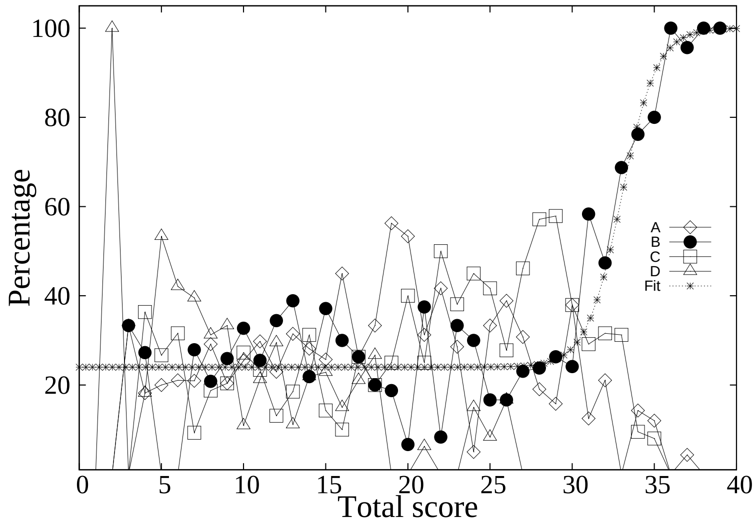

Figure 7.20: Item 19 - Item Response Curves

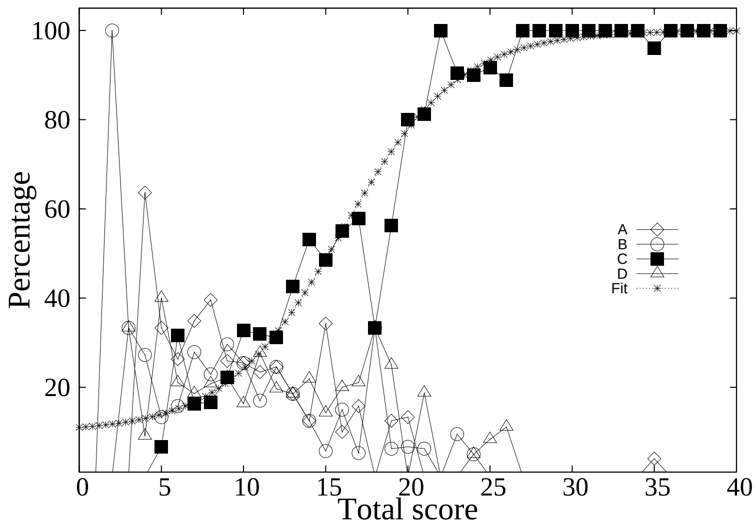

Figure 7.21: Item 20 - Item Response Curves

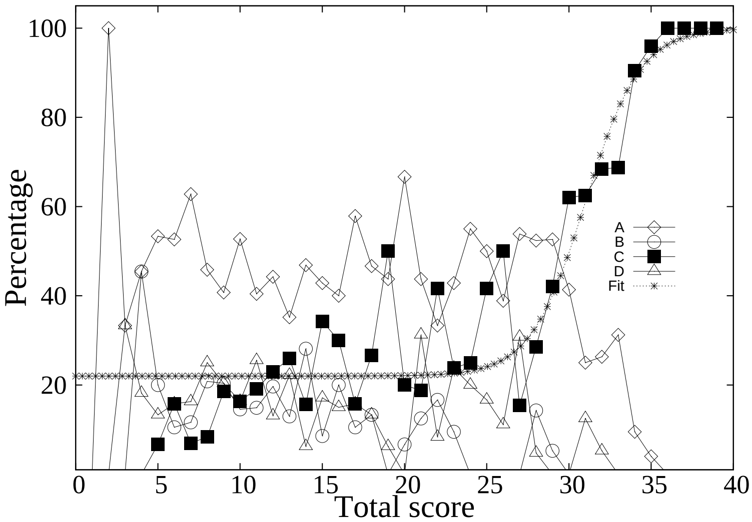

Figure 7.22: Item 21 - Item Response Curves

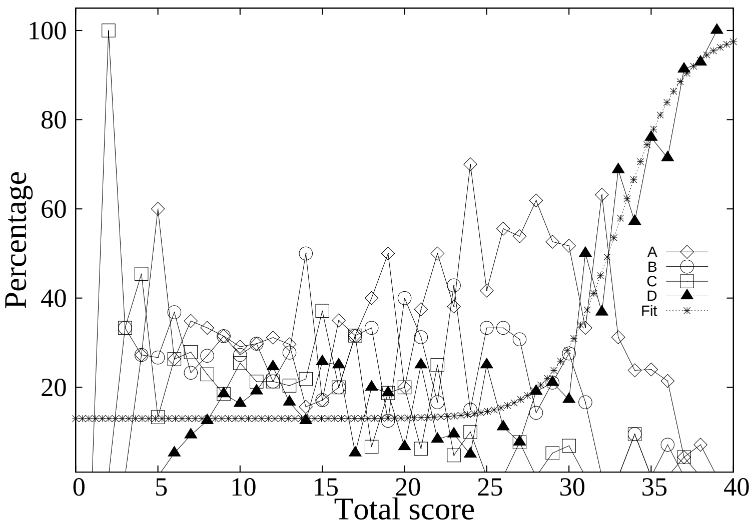

Figure 7.23: Item 22 - Item Response Curves

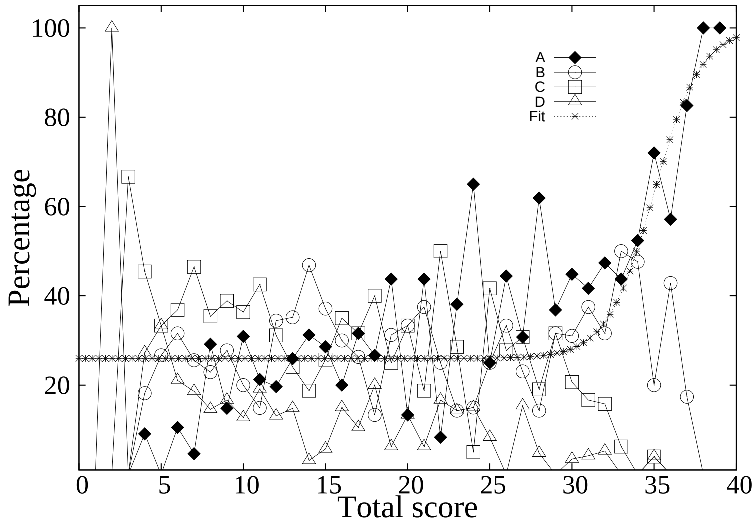

Figure 7.24: Item 23 - Item Response Curves

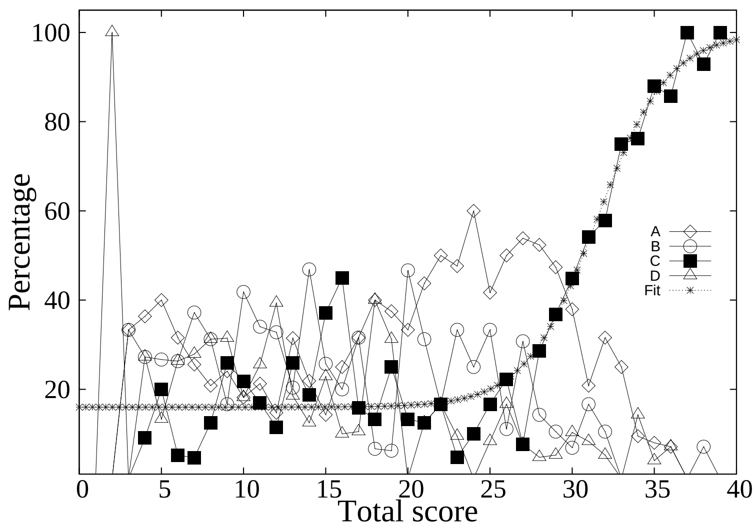

Figure 7.25: Item 24 - Item Response Curves

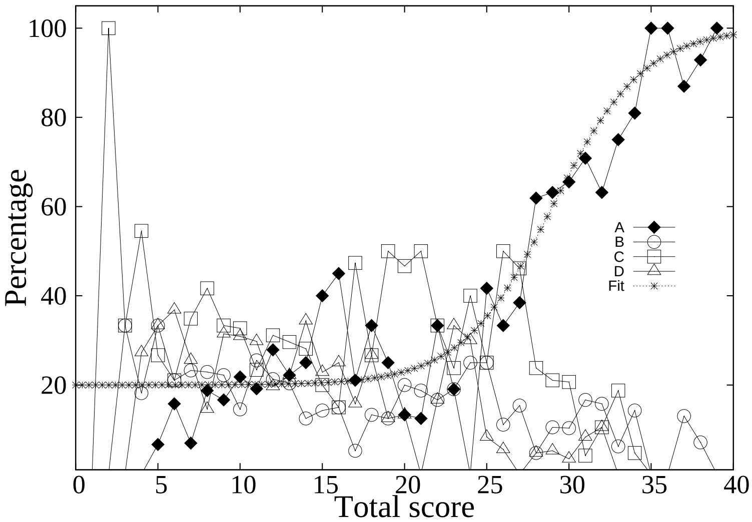

Figure 7.26: Item 25 - Item Response Curves

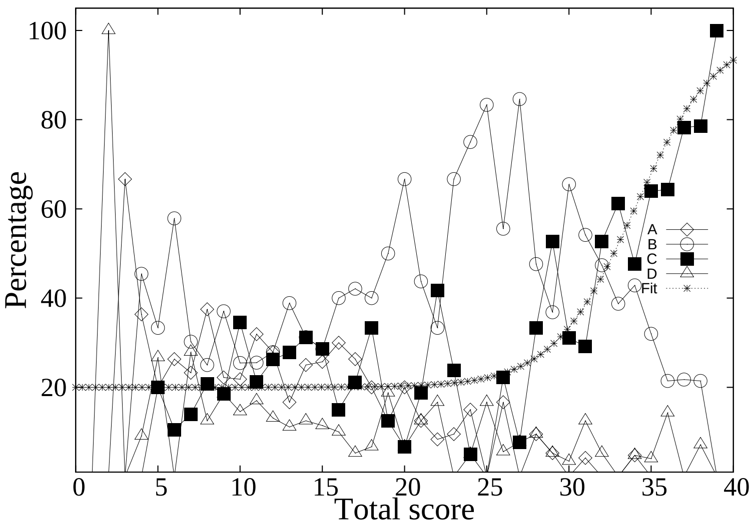

Figure 7.27: Item 26 - Item Response Curves

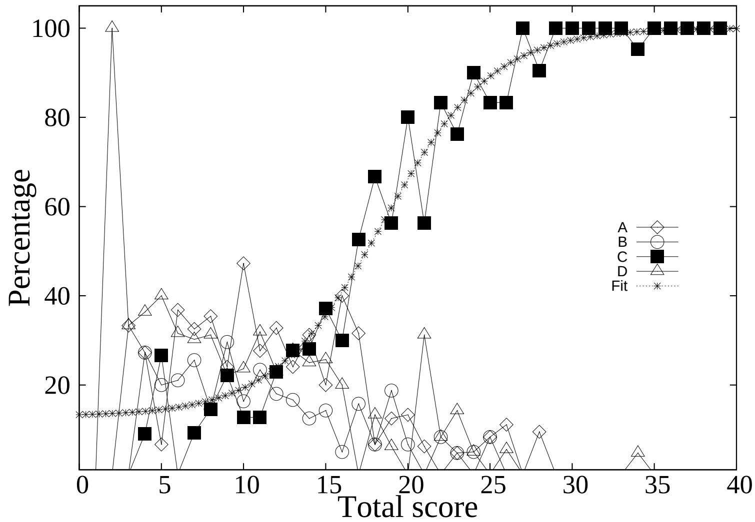

Figure 7.28: Item 27 - Item Response Curves

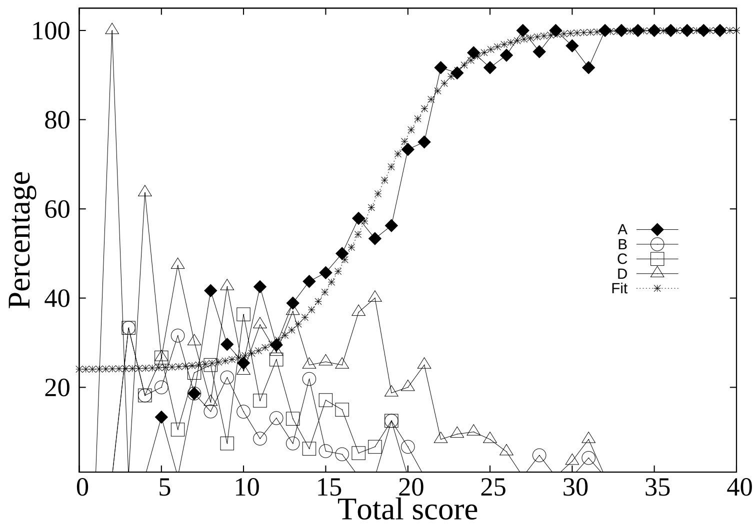

Figure 7.29: Item 28 - Item Response Curves

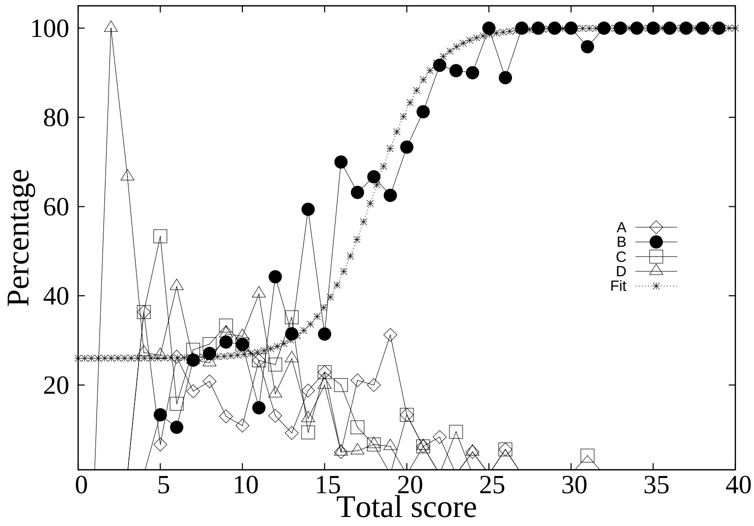

Figure 7.30: Item 29 - Item Response Curves

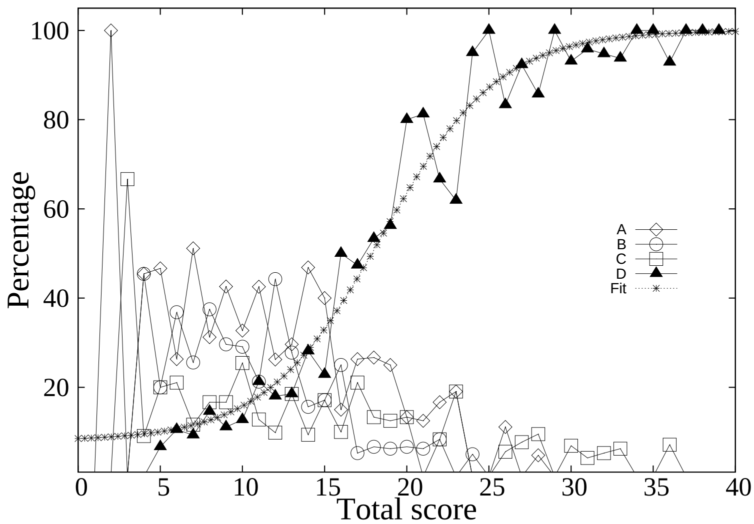

Figure 7.31: Item 30 - Item Response Curves

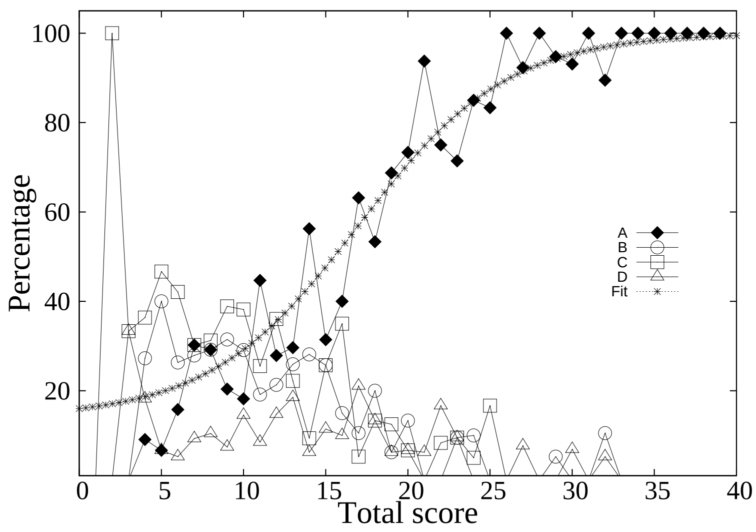

Figure 7.32: Item 31 - Item Response Curves

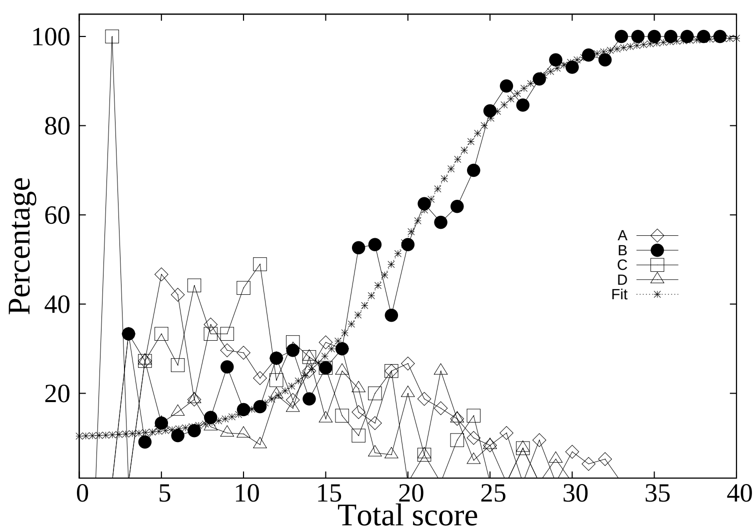

Figure 7.33: Item 32 - Item Response Curves

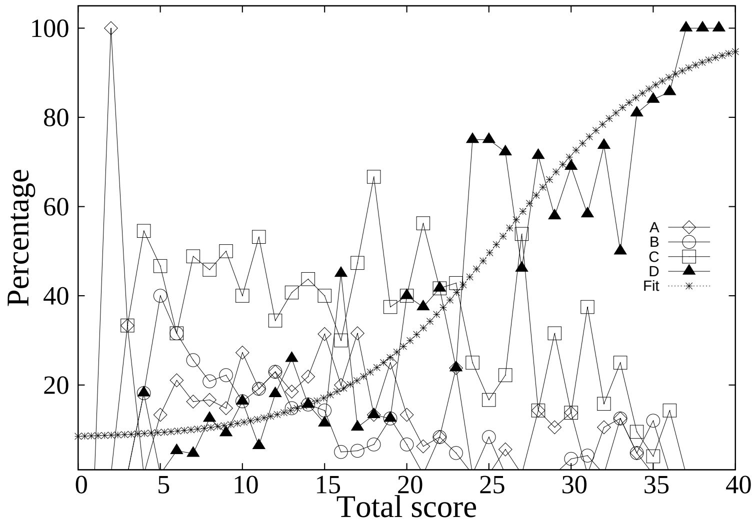

Figure 7.34: Item 33 - Item Response Curves

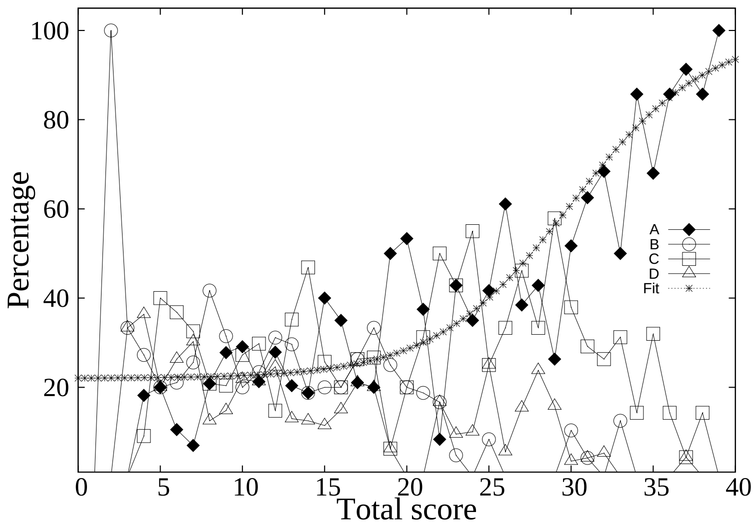

Figure 7.35: Item 34 - Item Response Curves

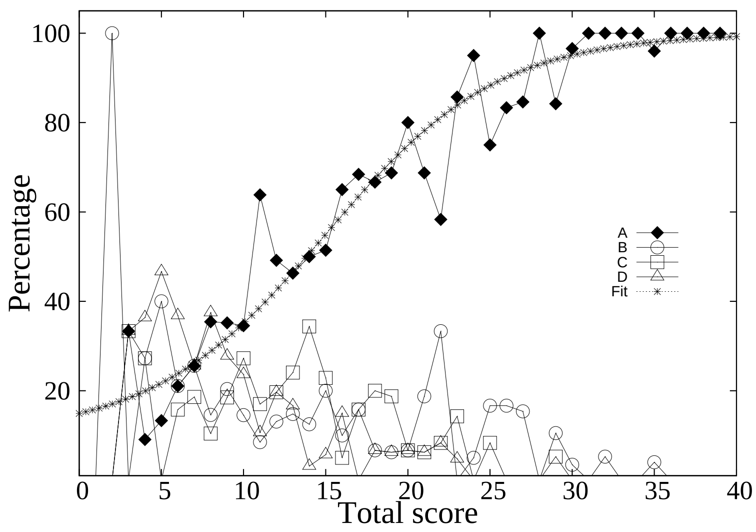

Figure 7.36: Item 35 - Item Response Curves

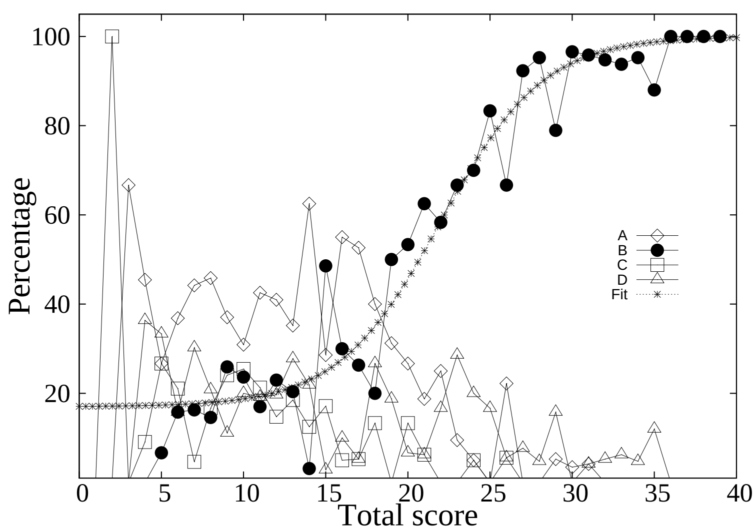

Figure 7.37: Item 36 - Item Response Curves

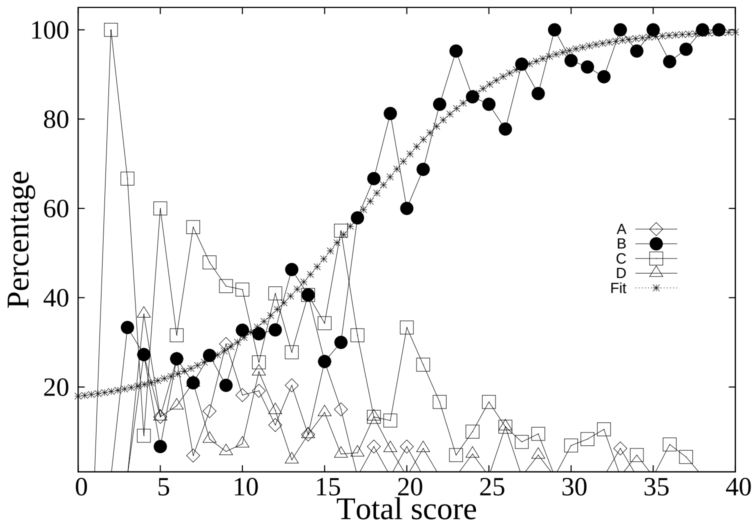

Figure 7.38: Item 37 - Item Response Curves

7.4 Concluding remarks

As may be recalled, the phase one sample was from schools in the Mumbai region. The sample size was relatively small. A larger and varied sample has made a deeper investigation of the items and the inventory as a whole possible. Consequently the conclusions and inferences will have their basis on a firmer footing. This was our motivation for undertaking the large scale administration in phase 2. The students (N=905) were from around 12 schools from 5 urban centers spread across India. They all had a course in rotational motion. The large sample size also enabled us to perform the IRC analysis.

The interview with the subset of students (N=35) further validated the items. We confirmed that the students interpreted the items and distractors as intended. We did not observe right choices being made from wrong reasoning. Some of the students displayed extremely poor understanding of rotational motion. For example a few were not even aware that angular velocity is a distinct concept from linear velocity. Such confused and vague understanding made the interview process difficult. In many cases students were not able to give any proper reasoning for their choices. They often based their choices on proximity to concepts of linear kinematics. Most

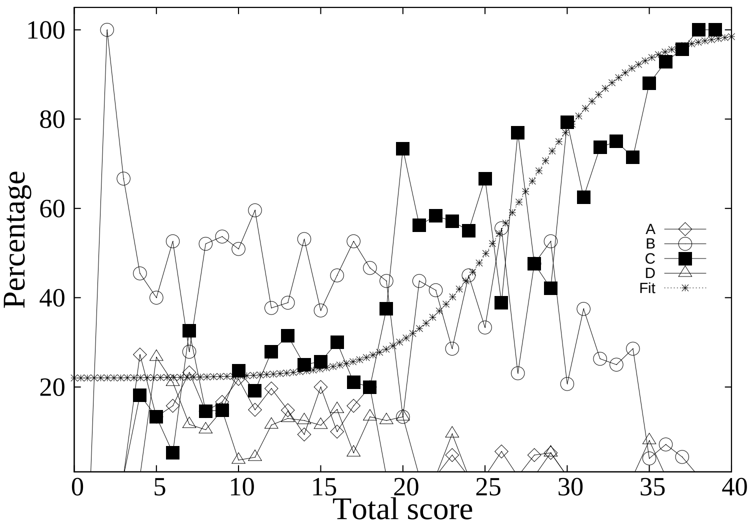

Figure 7.39: Item 38 - Item Response Curves

of our items had such distractors.

The response pattern of the students in phase 2 is broadly consistent with that of phase 1. It establishes the reproducibility of the broad patterns of thinking we identified in phase 1. We have mentioned these patterns in section 7.3 on IRCs. To recapitulate, these patterns include fixation with inappropriate prototypes (items 4, 5, 12, 13 and 19), indiscriminate use of equations (items 6, 19, 23, 36 and 37), pitfalls paralleling those found earlier in linear kinematics (items 3, 7-11 and 14-18) inappropriate extension of familiar procedural practices (items 26 and 27) , reasoning cued by primitive elements in thought (items 26, 27 and 28) and lack of differentiation between distinct but related concepts (items 5, 29, 30, 31 and 32). The large sample size of phase 2 put these inferences on a more solid footing. The interviews verified the existence of these patterns in student thinking. The similarity in response patterns to some of the questions mentioned above with a group of students at the University of Washington was discussed in chapter 6.

Phase 2 permitted comprehensive analysis of each item. The values of difficulty level to items in phase 2 were found to be higher compared to phase 1. Tables 7.2 and 7.3 help one identify items according to their difficulty, discriminatory power or consistency with the whole test based on the values P, D or r_{pbs} respectively. For example item 5 was moderately difficult (P=0.5), well discriminating (D=0.88) and

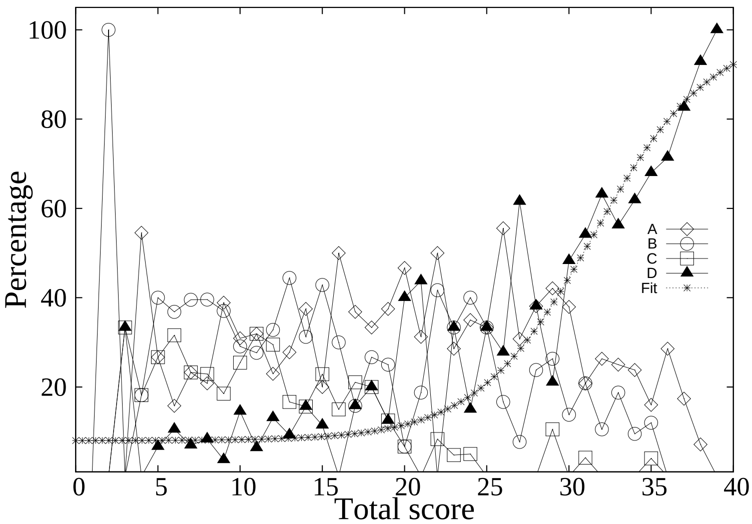

Figure 7.40: Item 39 - Item Response Curves

highly consistent with the whole test (r_{pbs} =0.72). The IRCs of the correct choice to all items had positive slope. They were clear sigmoids except for a few items (e. g., items 3, 7, 8, 10) Also most distractors are chosen by a significant portion of students. IRCs provide a visual display of most of the attributes conveyed by the indices P, D and r_{pbs}. In addition IRCs help us to identify the ability range of students to whom a particular distractor is appealing and this cannot be inferred from tables 7.2 and 7.3.

Another instance where IRC supplemented the analysis based on indices was with respect to items 6, 22 and 39. These items, as can be seen from tables 7.2 and 7.3 , have some of the indices below their desired values. However further analysis of the items based on the IRCs indicated that they were useful. The curves to the correct answers were sigmoids and the distractors were discriminating. Also there was no error in the content matter of the items. Thus on the combined basis of statistical and item response curve analysis we conclude that no items need to be dropped. The values of Kuder Richardson reliability index (r_{test}) and Ferguson's delta (δ) further affirms the quality of the test. They are well above their respective desired values.

Nevertheless, we provide the following suggestions for an instructor who wishes to employ a shorter version of the inventory or would like to tailor the test for a particular student sample. Tables 7.2 and 7.3 and IRCs (figures 7.2 to 7.40) can provide clear guidelines in this connection. For a class of moderate ability students the instructor may drop highly difficult items like 6, 22, 23, 26 and 39. Note that this includes items pertaining to the rotational kinematics of a particle in rectilinear motion (items 19-25) which are relatively difficult. Similarly, easier questions like 1 and 35 may be excluded if the students are of high ability. In short since we have delineated the characteristics of all items in detail instructors can choose the items based on their needs. Another suggestion to reduce the number of items in the inventory concerns items 7-11 and 14-18 which probe angular velocity and angular acceleration separately in 5 identical contexts involving a simple pendulum. These 2 pairs of items can be clubbed together reducing the number of items from 10 to 5.

Chapter 8

Conclusion

8.1 Summary

Rotational motion is arguably one of the most difficult topics in higher secondary school level physics, both for students as well as teachers. Our numerous interactions with students and teachers have confirmed this. Despite its difficult nature, it has not attracted the attention of physics education research community for a long time. Rimoldini and Singh (2005) initiated a systematic research on the topic with their broad survey instrument . The work by Ortiz et al (2005) during the same period was also a step in the same direction. Both these works reveal the conceptual richness of the topic and the research potential it holds. Our own experience with students as well as teachers laid bare an array of difficulties in this area. Our first impression was that the difficulties lay in relatively advanced aspects like dynamics of rolling motion. This led us to investigate student understanding of the direction of friction on rolling bodies (Singh and Pathak, 2007). However, further interactions with students revealed that the roots of the problem lay in a shaky foundation. We noted that even elementary concepts like angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$) presented serious difficulties to students as well as teachers. This resulted in our present work on rotational kinematics. We were led to develop an inventory focusing on the concepts of angular velocity and angular acceleration.

Our decision to develop concept inventories (CI) was motivated by the need to provide momentum to science education research in the country. There is a stark necessity for research driven reforms in all aspects spanning from assessment to teacher professional development. We maintain that CIs can play a significant role in efforts in these directions. Such a standpoint is supported by the enabling impact inventories had on physics education research in US (Richardson, 2004; Hake, 2011). CIs hold potential beyond their direct utility as a ready use diagnostic and assessment tool for instructors. The inventory items can serve as clicker questions which constitute a key element of the Peer Instruction pedagogy (Mazur, 2007). It may be noted that Peer Instruction is currently among the most popular research driven pedagogies employed globally. CIs also have the potential to be a platform for teacher professional development. This is best illustrated by looking into the development of the Force Concept Inventory (FCI). It is seldom noticed that the two authors of FCI besides Hestenes were high school instructors. Development of FCI indicates one way in which the scientific community in our universities can play a useful role in capacity building of teachers in schools and colleges. CIs can play a crucial role in large scale science education reform initiatives as exemplified by the Carl Wieman Science Education Initiative (Wieman, 2007; Adams and Wieman, 2010). Inventories facilitate an objective assessment of curricula and pedagogies. A scientific approach to science education is enabled by reliable large scale assessments.

We expect that our modest effort towards construction of concept inventories, in addition to their direct utility, is a positive step towards the greater possibilities mentioned in the previous paragraph. Keeping this aspect in mind we ensured that all the steps and salient features involved in the construction of our inventory were documented (see chapter 2). The documentation can provide guidelines to others interested in similar endeavors in Indian context. The systematic and iterative aspect of the process of construction of inventories is to be particularly noted. The development of our inventory began with a theoretical analysis of text book presentations of rotational kinematics. Prototypical problems given at the end of the textbooks and those employed in competitive examinations like the Indian Institute of Technology - Joint Entrance Examination were reviewed. We carried out a cognitive analysis of $\vec{\omega}$ and $\vec{\alpha}$ akin to that carried out by Reif and Allen (1992) for linear acceleration. These theoretical analyses were followed by extensive empirical studies. Questions in the initial phases involved open ended and explanatory type among others. Verbal data was collected using think aloud protocol, retrospective probing and interviews

(Young, 2005; Rimoldini and Singh, 2006; Adams and Wieman, 2010). Interaction with students, teachers and experts constituted the core of the development process. Insights from these interactions fed into the construction of appropriate items and distractors. The items were refined after pilot tests. Items and distractors were successively modified to incorporate insights obtained from student responses. New items were added to ensure that all aspects of the concepts were covered. In total around 50 students participated in this initial phase of development of the inventory (Mashood and Singh, 2013). The inventory was further pilot tested with a group of undergraduate students (N=58). They were asked to write down brief explanations for their answer choices.

Retrospectively, the inventory developed in three parts, namely rotational kinematics of a particle (see chapter 3), special case of a particle in rectilinear motion (see chapter 4) and rotational kinematics of a rigid body about a fixed axis (see chapter 6). The development followed the same order as mentioned. The test underwent iterative cycles to evolve into the current form constituting 39 items. Most of the items are qualitative. A few are semi-quantitative The knowledge of calculus and algebraic manipulations required for answering the items, if any, are minimal. Throughout the development of the inventory we have been careful in this respect. A case in point is question 22 which dealt with the angular acceleration of a particle in rectilinear motion. The magnitude of angular acceleration exhibits non-monotonic behavior, increasing in the beginning, allowing a maximum and then decreasing. This non-monotonic behavior was not probed in the inventory, since as apparent from chapter 5, it involved considerable algebraic manipulations. We were careful to limit ourselves to the case of large distance where angular acceleration decreases monotonously. This could be understood purely on the basis of asymptotic reasoning.

The items were content validated by experts and face validated by teachers and students throughout the process. The phase one of the administration of the inventory constituted two groups of higher secondary school students (N=79 and 74) and two groups of teachers (N = 26 and 25). One group among higher secondary students (N=74) was the olympiad aspirants which represented a high ability group. The teachers taught physics at the higher secondary or undergraduate level. They hailed from different parts of the country. We asked the students to write brief explanations for their answer choices. Insights from this fed into the inventory. The frequency with which distractors were chosen were analyzed. We made inferences about the the patterns of thinking prevalent among students. The analysis was supplemented by semi-structured interviews with a subset of the students (N=6).

The broad patterns of thinking among students we uncovered include fixation with inappropriate prototypes, indiscriminate use of equations, pitfalls paralleling those found earlier in linear kinematics, inappropriate extensions of familiar procedural practices, reasoning cued by primitive elements in thought, and lack of differentiation between related but distinct concepts. We connected these findings with patterns of thinking identified by researchers in other topics and these are discussed in detail in chapters 3, 4 and 6. An illustrative example is the instance indicating reasoning cued by primitive elements in thought. Close and Heron (2011) found a case of the pattern in the context of student thinking regarding the concept of energy. They observed that students in general tend to prefer conservation of energy over other conservation laws because the former resonates with certain primitive elements in their thinking. We noted a similar reasoning pattern among students wherein the concept of center of mass enjoyed a special status in student minds. The identifications of broader patterns of thinking like the one mentioned here is important. This is because of the fact that it indicates that student errors share certain general features spanning across topics. Administration of part of the inventory to students (N=384) at the University of Washington, Seattle revealed that most of the distractors indicative of these patterns were popular among American students as well.

In order to verify the patterns of thinking we identified in phase 1, further validate the items, and subject the test to detailed statistical scrutinies, we undertook a large scale administration of the inventory (see chapter 7). The sample comprised of higher secondary students (N=905) from 5 urban centers (Jaipur, Patna, Mumbai, Hyderabad and Bangalore) spread across the country. A subset (N=35) of students were interviewed. The interviews verified the patterns of thinking we uncovered during earlier administrations. We carried out both item wise and whole test analyses using widely employed statistical indices (Ding et al., 2006; Ding and Biechner, 2009). The item-wise statistics we calculated were the difficulty level, index of discrimination and the point biserial coefficient. The values of these indices were well above their desired values for all items with few exceptions. This substantiates the quality of the items. The average difficulty level, index of discrimination and point biserial coefficient for the 39 items were 0.48, 0.65 and 0.63 respectively. In addition the indices provide clear guidelines about the level of difficulty of the items, their discriminatory power and consistency with the whole test. This will enable instructors to make an informed choice among the items based on their particular needs.

As we have mentioned earlier the items were validated at various phases of the study. Experts examined the content validity. Validation interviews with students facilitated face validity. Equally important is to establish the reliability of the inventory. Administration of part of the inventory at the University of Washington and the subsequent consistency in student responses indicated the reproducibility of some of our results. Nevertheless the importance of quantitatively establishing reliability led us to calculate the Kuder Richardson reliability index for the inventory. We also calculated the Ferguson's delta. These are given in table 8.1. Both are well above their desired values indicating the reliability and discriminatory power of the test. In the large scale administration we also administered the Force Concept Inventory (FCI) and the Conceptual Survey on Electricity and Magnetism (CSEM). These inventories are standardized and prominent among inventories in PER. FCI comprises of 30 items on basic mechanics. CSEM has 32 items on electricity and magnetism. The CSEM was administered to a subset (N=554) of the larger sample (S3, N=905) since only these students had been taught electricity and magnetism. The Kuder Richardson reliability index and the Ferguson's Delta were also calculated for FCI and CSEM. The values of the indices are given in table 8.1. As can be seen the values for our inventory are consistent with those obtained for FCI and CSEM.

In addition to the statistical analysis, we also carried out item response curve (IRC) analysis. It may be noted that it was the large sample size (N=905) of phase 2 which made the IRC analysis possible. IRCs to all 39 items were plotted. The curves to the correct choice to all items have positive slope and correlated positively with the ability level. Most of the distractors were chosen by a significant section of candidates indicated by peaks in their corresponding curves. IRCs provided a visual confirmation of the insights obtained by the statistical analyses. The curves vividly

Table 8.1: The average score (in percentage), Kuder Richardson reliability index and Ferguson's Delta for FCI, CSEM and our inventory on rotational kinematics (RK), along with the sample size for each test.

	Desired values	FCI	CSEM	RK
Sample size (N)		915	554	905
Average score		51.36 %	46.06 %	47.18 %
Kuder Richardson reliability index	≥ 0.8	0.95	0.96	0.93
Ferguson's delta	≥ 0.9	0.99	0.98	0.99

displayed the level of difficulty of the items and their discriminatory power. More importantly they revealed insights provided neither by tabulation of the frequency of choices to each item nor the statistical indices we employed. This was the range of the ability level of students to whom particular distractors to an item appealed. IRCs also helped us further investigate the quality of a few items for which the indices were problematic. The IRCs to these items revealed their useful characteristics which otherwise would have been dropped. Despite being highly informative it is unfortunate that IRCs are used sparingly in PER. This is inspite of the fact that they are no longer computationally demanding. The statistical and IRC analyses delineated the characteristics of all items in detail. This should enable instructors to choose the items based on their needs in classrooms.

The results from our studies were submitted to international peer reviewed journals in physics education research at appropriate stages of our study. The reviews from the referees provided valuable insights and critical inputs. The publication of the results ensured that the research was proceeding in the right direction. It also situated our work in the broader discourse among the research community. Other researchers have been pursuing studies closely related to our work. Notable among them was the work by Close and Heron (2011) on angular momentum which happened around the same period as our study. This study investigated student understanding of angular momentum of a particle moving in a straight line.

8.2 Pedagogical implications

We briefly discuss some of the pedagogical implications of our work. Our study revealed that students as well as teachers experience an array of difficulties in rotational kinematics of a particle. This may be because rotational kinematics of a particle is inadequately dealt with in most text books (Mashood and Singh, 2012c). We suggest emphasizing operational definitions and procedural specifications as immediate corrective measures. We have provided an operational definition for angular velocity of a particle (Mashood and Singh, 2012b). Our work as well as the earlier study by Rimoldini and Singh (2005) indicates that part of the difficulties can be traced to related difficulties in basic mechanics. This suggests that discussion of linear and rotational concepts in the same context will help students to better differentiate between them. The oscillating simple pendulum is a rich context for such a purpose. Student understanding of linear velocity and acceleration in the context of a simple pendulum had been probed earlier (Reif and Allen, 1992; Shaffer and McDermott, 2005). Our inventory comprises of a set of five questions each on angular velocity ($\vec{\omega}$) and angular acceleration ($\vec{\alpha}$) posed in the same context. These concepts are probed with the bob at the extreme, mean and an intermediate position. Two items concern the variation of $\vec{\omega}$ and $\vec{\alpha}$ as the pendulum swings. With the help of these items we uncovered a range of pitfalls and misconceptions harbored by students regarding $\vec{\omega}$ and $\vec{\alpha}$. Details of our findings can be found in chapter 3 (Mashood and Singh, 2012b). The fact that students can easily design a real pendulum is an added educational advantage. Taking into account the pedagogical richness we suggest using simple pendulum as a context to teach both linear and rotational kinematics. It may be noted that none of the popular textbooks has such a discussion (Halliday et al., 2001; Young and Freedman, 2004; Giancoli, 2005; NCERT, 2006).

We suggest another pedagogically rich context to discuss kinematics namely a particle in rectilinear motion. The particle moving in a straight line with constant velocity is perhaps the simplest type of motion. Our study revealed an array of difficulties harbored by students pertaining to rotational kinematics in this context. Details of our findings can be found in chapter 4 (Mashood and Singh, 2012b). Our focus on linear kinematics in this context was limited to what is directly relevant to our discussion on rotation. We probed student understanding of the variation of the radial and azimuthal components of linear velocity of the particle. The context affords possibilities for probing other aspects of linear kinematics. In addition to the analytical approach, we encourage use of graphs to approach these problems. One of the major pre-requisites in dealing efficiently with kinematics (linear and rotational) is familiarity with vector operations. Students can draw position vectors of the particle on a graph for successive instants. They can then perform vector subtraction and examine the behavior of the angular variable as the position vector changes.

8.3 Limitations of concept inventories

We discuss some of the concerns raised by concept inventories. First among them is the ability of multiple choice questions to accommodate conceptual nuances and intricacies. The extend to which they can be covered by a limited number of simple statements (the distractors) is debatable. Further, the items are closed-ended (Smith and Tanner, 2010). Students are forced to choose one among the four or five options available. If all possible modes of responses are not represented by the distractors the test may be imposing an answer on the student. Students may also answer by resorting to logical tricks without engaging with the subject content. There can be cases of false negatives and false positives (Hestenes and Halloun, 1995). Another limitation is that we do not know a students' thinking in detail while she is making an answer choice to an item. This can be remedied if we interview the student. As such interviews are always encouraged (Adams and Wieman, 2010).

It may be possible that distractors to items relevant to a population may not be appropriate for another. Administration of part of the inventory at the University of Washington revealed an instance of this. It was found that there exists differences between the American and Indian students when it came to the direction of angular velocity. The differences seemed to be on account of the variation in the instruction in India and US. These differences in student thinking need to be taken into account. The context dependence of novice thinking is another issue of concern related to CI's (Bao and Redish, 2006). Novices may correctly answer a question about a concept in one context and incorrectly in another. As a result, when a student answers an item it becomes difficult to delineate the effect of understanding from the effect of context. It has been shown that the timing of administration of a CI and the incentives for the participants can affect the results (Ding et al., 2008). It is important that factors like these are noted. What precisely a concept inventory measures is another issue of concern. This has been extensively debated in the case of the Force Concept Inventory (Huffman and Heller, 1995). As per the critics, FCI rather than measuring an understanding of the force concept, is more likely measuring 'bits and pieces of student's knowledge that do not necessarily form a coherent force concept'. So apart from the phenomenological insights provided, a cognitive science perspective of what CI's actually measure is unclear.

We were aware of these limitations and have attempted to address them at least partially. The inventory was constructed systematically and iteratively. We interviewed a cross section of students. Brief explanations were sought from students during pilot studies. Thus considerable care was taken to develop the distractors so that they represent students' thinking. Efforts were made to structure items so that they are not answerable by resorting to logical tricks. We limited our inventory to kinematic concepts alone. Such a focused approach helps narrow down the conceptual difficulties experienced by students. This would be difficult if items involved greater number of concepts or involve considerable mathematical manipulations. It may be recalled that, despite these limitations, an inventory has its advantages, which we have discussed in chapter 1. They include its ready to use characteristic, possibility of easy and rapid evaluation and the potential for large scale application.

8.4 Future directions

We conclude by mentioning some of the possible avenues that our work opens up. Quality science education to a larger populace is important for the economic and societal progress of the country. We need to promote research driven education reforms to achieve this. Our work points to the stark necessity of meaningful teacher professional development in the country. One possible way to achieve this is to ensure scientific communities in our universities collaborate with practicing teachers in developing CIs. Rotational kinematics of a particle needs to be dealt with in detail before addressing rigid bodies. We think that such a sequencing would be pedagogically more prudent (Mashood and Singh, 2012a). We need to promote a culture of learning where alternative conceptions and pitfalls are considered as resources to achieve a better understanding. Instructors who are in a state of denial regarding the findings of science education research need to be made aware of the importance of research in education. CIs can play a facilitating role in this regard. Interactive pedagogies like peer instruction (Mazur, 2007) should supplement traditional modes of instruction. CI items can be adapted as clicker questions. Furthermore research driven curriculum like the Tutorials in Introductory Physics (McDermott et al., 2002) needs to be developed to address pitfalls in understanding. In all these efforts a synergistic collaboration of practicing teachers, scientific communities in the universities and researchers in education is important. We have made modest efforts in this regard. Further, analysis of the misconceptions and pitfalls we identified from the perspective of cognitive science would be fruitful. Such an endeavor would throw light on why these misconceptions and pitfalls arise. It may then help us to address them better.

In future we plan to administer the test to varying samples which will extend to rural India. It is possible that we will uncover differences between the urban and the rural samples. Administering the test to rural India brings additional challenges. The varying culture and languages of rural India makes the process a daunting task. It will require translation to local languages and use of local idioms. We have initiated our effort in the national language Hindi. Inventory construction is an iterative process. As such we are aware that even the present form of the test is open to refinement and modifications. So far we have covered only angular velocity and angular acceleration . We did not dwell into more complex aspects like rolling with or without slipping. These areas along with other concepts like conservation of angular momentum and torque-angular momentum relation hold potential for future research which one can undertake. It is encouraging to know that other researchers are pursuing related areas. Biechner is currently working on student understanding of graphs in the context of rotational motion (private communication). The physics education group at the university of Washington intends to revise the chapter on rotational motion in their Tutorials in Introductory physics (McDermott et al. 2002). We also plan to work on an Indian adaptation of the same in future.

Appendix A

Questionnaire on Rotational Kinematics

A.1 Instructions

Each question or incomplete sentence is followed by four suggested answers or completions. Select the **one** that is the most appropriate in each case.

A.2 Rotational kinematics of a particle

A.2.1 Angular Velocity of a Particle

Questions 1–3 are concerned with figure A.1. The figure depicts a wall clock showing the second hand and minute hand (the hour hand is not shown). A and C denotes the tip of the second hand and minute hand respectively. The point B on the second hand is at the halfway distance, namely OA = 2OB. The clock is in the x-y plane with the z axis coming out of the plane of the paper. The unit vectors in the x, y, z direction are $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ respectively. We consider angular velocity about O.

1. Let ω_{A}, ω_{B} and ω_{C} be the angular speeds of points A, B and C respectively. Which of the following is the correct relation?

   (a) $\omega_{\mathrm{A}} > \omega_{\mathrm{C}} > \omega_{\mathrm{B}}$

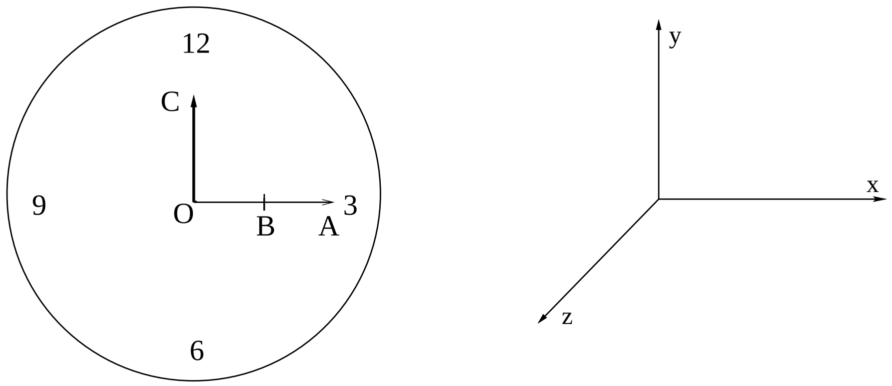

Figure A.1: Wall clock (see questions 1-3).

(b) $\omega_\mathrm{A} = \omega_\mathrm{B} = \omega_\mathrm{C}$

(c) $\omega_\mathrm{A} = \omega_\mathrm{B} > \omega_\mathrm{C}$

(d) $\omega_\mathrm{A} > \omega_\mathrm{B} = \omega_\mathrm{C}$

2. The clock is transparent and the second hand can be viewed from both the front and the back side. Then,

    (a) if viewed from the front, the second hand moves anticlockwise with angular velocity in the - $\hat{k}$ direction.

    (b) if viewed from the front, the second hand moves clockwise with angular velocity in the $\hat{k}$ direction.

    (c) if viewed from the back, the second hand moves anticlockwise with angular velocity in the $\hat{k}$ direction.

    (d) if viewed from the back, the second hand moves anticlockwise with angular velocity in the - $\hat{k}$ direction.

3. At an instant t, both the second hand and the minute hand of the clock are at 12 O'clock position. Regarding the angular speeds of A and C at time t which of the following statements is true ?

    (a) Angular speed of A is greater than the angular speed of C.

    (b) Angular speed of A is less than the angular speed of C.

    (c) Angular speed of A is equal to the angular speed of C.

(d) Angular speed at an instant cannot be defined.

4. A particle is moving in a circle about the origin O with **increasing speed** as shown in figure A.2. At two points A and B on the circumference of the circle, the angular velocity $\vec{\omega}$ about O

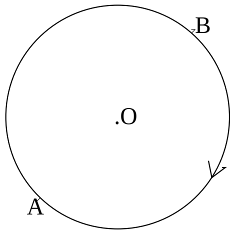

Figure A.2: Particle in circular motion (see questions 4, 12 and 13).

(a) differs only in magnitude.
(b) differs only in direction.
(c) differs in both magnitude and direction.
(d) is the same in magnitude and direction.

Questions 5–6 are concerned with figure A.3. A planet is revolving in an elliptical orbit with sun at one of its foci. A and B are two distinct points on the trajectory of planet. We consider angular velocity of the planet about the sun regarded fixed.

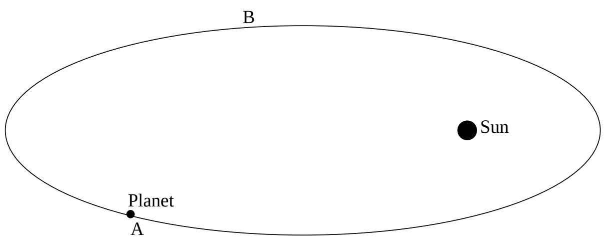

Figure A.3: Planet moving in an elliptical orbit with sun at one of its foci (see questions 5 and 6).

5. The direction of the angular velocity vector of the planet at point A, $\vec{\omega}_A$ is

(a) radially inward towards sun.

(b) inward but not radial.

(c) tangential to the orbit at the point A.

(d) perpendicular to the plane of motion.

6. Let $\vec{v}_{\mathrm{B}}$ be the linear velocity, $\vec{\omega}_{\mathrm{B}}$ the angular velocity and $\vec{r}_{\mathrm{B}}$ the position vector of the planet at B (all about the sun). Which of the following statements is correct?

   (a) $\vec{v}_{\mathrm{B}} = \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ from the definition of angular velocity.

   (b) $\vec{v}_{\mathrm{B}} = \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because the planet is in rotational motion.

   (c) $\vec{v}_{\mathrm{B}} \neq \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because the motion is not circular.

   (d) $\vec{v}_{\mathrm{B}} \neq \vec{\omega}_{\mathrm{B}} \times \vec{r}_{\mathrm{B}}$ because $\vec{\omega}_{\mathrm{B}}$ is not perpendicular to the plane of motion.

Questions 7–11 are concerned with figure A.4. Figure shows a simple pendulum oscillating about the mean position B. A and D are the left and right extreme positions respectively. Consider the plane of the paper to be the plane of motion of the bob. We consider angular velocity about O.

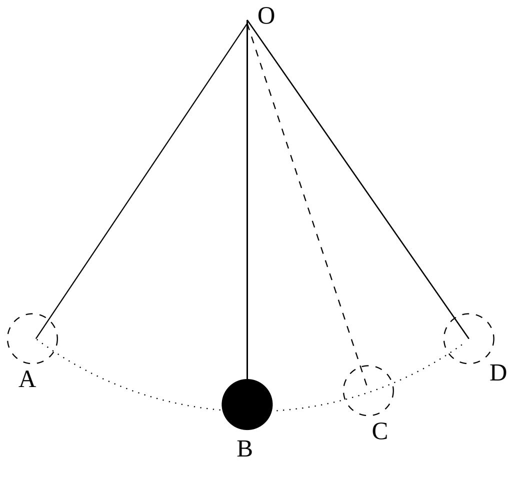

Figure A.4: Oscillating simple pendulum (see questions 7-11 and 14-18).

7. Regarding the angular velocity of the bob at the instant when it is in the left extreme position A, which of the following statements is true?

(a) Angular velocity is zero.

(b) Angular velocity at a single instant is undefined.

(c) Angular velocity at a single position is undefined.

(d) Angular velocity is non zero.

8. Angular velocity of the bob at the mean position B is

(a) non zero and radially inward.

(b) zero.

(c) the maximum (in magnitude).

(d) non zero (but not the maximum) and perpendicular to the plane of motion.

9. As the pendulum moves from A to D, the direction of angular velocity of the bob between A and D

(a) is in the plane of motion and remains the same.

(b) is in the plane of motion and keeps changing.

(c) is perpendicular to the plane of motion and remains the same.

(d) is perpendicular to the plane of motion and flips at the mean position.

10. Throughout the motion of the bob from A to D the magnitude of the angular velocity of the bob

(a) remains same.

(b) keeps on increasing.

(c) first increases and then decreases.

(d) first decreases and then increases.

11. The angular velocities of the bob at a point C on the trajectory, when going from A to D and from D to A are

(a) equal in magnitude, but differ in direction.

(b) the same in direction, but differ in magnitude.

(c) different in both magnitude and direction.

(d) equal in both magnitude and direction.

A.2.2 Angular Acceleration of a Particle

Questions 12–13 are concerned with figure A.2. Figure shows a particle moving clockwise in a circle about the center O with **increasing speed**. A and B are two distinct points on the trajectory of the particle. We consider angular acceleration about O.

12. The angular acceleration of the particle at point A is

    (a) zero.

    (b) non zero because only the direction of angular velocity keeps on changing.

    (c) non zero because only the magnitude of angular velocity keeps on changing.

    (d) non zero because both the magnitude and direction of angular velocity keep on changing.

13. The directions of angular acceleration of the particle at two distinct points A and B on the circle

    (a) are the same and out of the plane of the paper.

    (b) are the same and into the plane of the paper.

    (c) are different.

    (d) are undefined since the magnitude of acceleration is zero.

Questions 14–18 are concerned with figure A.4. Figure shows a simple pendulum oscillating about the mean position B. A and D are the left and right extreme positions respectively. Consider the plane of the paper to be the plane of motion of the bob. We consider angular acceleration about O.

14. Regarding the angular acceleration of the bob at the instant when it is in the extreme position A on the left, which of the following statements is true?
    (a) Angular acceleration is zero.
    (b) Angular acceleration at a single instant is undefined.
    (c) Angular acceleration at a single position is undefined.
    (d) Angular acceleration is non zero.
15. Angular acceleration of the bob at the mean position B is
    (a) non zero and radially inward.
    (b) zero.
    (c) the maximum (in magnitude).
    (d) non zero and perpendicular to the plane of motion.
16. As the pendulum moves from A to D, the direction of angular acceleration of the bob between A and D
    (a) is in the plane of motion and remains the same.
    (b) is in the plane of motion and keeps on changing.
    (c) is perpendicular to the plane of motion and remains the same.
    (d) is perpendicular to the plane of motion and changes the direction.
17. Throughout the motion of the bob from A to D, the magnitude of the angular acceleration of the bob
    (a) remains the same.
    (b) keeps on increasing.
    (c) first increases and then decreases.
    (d) first decreases and then increases.
18. The angular accelerations of the bob at a point C on the trajectory, when going from A to D and from D to A are

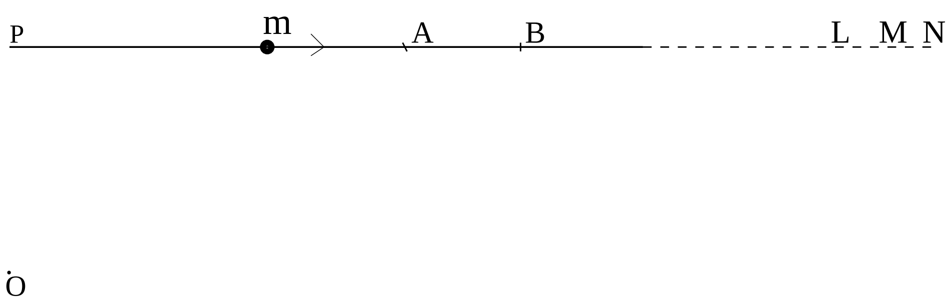

Figure A.5: Particle moving in a line (see questions 19-25).

(a) equal in magnitude, but differ in direction.

(b) the same in direction, but differ in magnitude.

(c) different in both magnitude and direction.

(d) equal in magnitude and direction.

A.3 Particle Moving in a Straight Line

Questions 19-25 are concerned with figure A.5. Figure shows a particle moving in a straight line with constant velocity. The origin O as shown in the figure is not on the line. We consider angular velocity and angular acceleration about O.

19. The magnitudes of angular velocities of the particle (about O) at points A and B are $\omega_{\rm A}$ and $\omega_{\rm B}$ respectively. Which of the following is true?

    (a) $\omega_{\rm A} > \omega_{\rm B}$ because $v = \omega r$, v is constant and $r_{\rm A} < r_{\rm B}$.

    (b) $\omega_{\rm A} > \omega_{\rm B}$ because the angular displacement at A is greater than angular displacement at B for the same interval of time.

    (c) $\omega_{\rm A} = \omega_{\rm B}$ because there is no acceleration.

    (d) $\omega_{\rm A} = \omega_{\rm B} = 0$ because the motion is linear.

20. The direction of angular velocity of the particle at A is

(a) along AB.

(b) along OA.

(c) perpendicular to the plane of the paper.

(d) undefined since its magnitude is zero.

21. The angular acceleration of the particle at point A on the line is

    (a) zero because the motion is linear.

    (b) non zero because angular velocity changes in direction only.

    (c) non zero because angular velocity changes in magnitude only.

    (d) non zero because angular velocity changes in both magnitude and direction.

22. Consider the motion of particle at large values of r, in the region LMN. As the particle moves from L to N the magnitude (absolute value) of angular acceleration of the particle

    (a) remains zero because the motion is linear.

    (b) is non zero and remains constant because there is no linear acceleration.

    (c) keeps increasing.

    (d) keeps decreasing.

23. Torque τ on the particle at any instant is

    (a) zero because force acting on the particle is zero.

    (b) non zero because there exists an angular acceleration.

    (c) zero because $\tau = I\alpha$ and $\alpha = 0$ (I is the moment of inertia and α is the angular acceleration).

    (d) undefined since the motion is linear.

24. The direction of angular acceleration is

    (a) undefined since its magnitude is zero.

(b) along the direction of angular velocity.

(c) opposite to the direction of angular velocity.

(d) along the direction of torque.

25. As the particle moves away from P, which of the following statements regarding linear velocity of the particle is correct?

(a) The radial component increases.

(b) The tangential (azimuthal) component increases.

(c) Both the radial and tangential (azimuthal) components remain constant since the velocity is constant.

(d) The radial component remains constant since the centripetal force is zero.

A.4 Rigid Body Rotation About a Fixed Axis

26. The magnitude of angular velocity ω of a rigid body rotating about a fixed axis is given by $\omega = \Delta\theta/\Delta t$. The angle $\Delta\theta$ here is the angle traced by

(a) the position vector of the center of mass of the rigid body (from a specified origin).

(b) the position vector of any particle on the body (from a specified origin).

(c) a line perpendicular to the axis from any particle on the body.

(d) a line perpendicular to the axis from the center of mass only.

Question 27 is concerned with figure A.6. Figure A.6 shows a pulley of radius R rotating about a fixed axis **with increasing angular speed.** The axis is passing through the center P. The angular velocity of the pulley is $\vec{\omega}$ and the angular acceleration is $\vec{\alpha}$.

27. The angular velocity of the pulley is

(a) the vector sum of angular velocities of all the particles constituting the pulley.

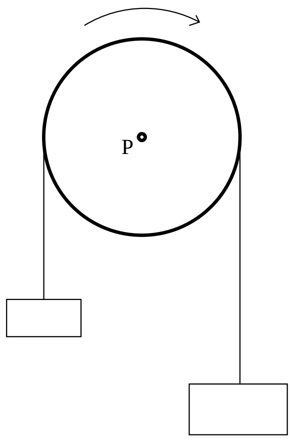

Figure A.6: A rotating pulley (see question 27).

(b) the scalar sum of the magnitude of angular velocities of all the particles constituting the pulley.

(c) equal to the angular velocity of any particle on the pulley.

(d) equal to the angular velocity of the center of mass of the pulley.

28. A rigid body is rotating about a fixed axis. Motion of an arbitrary particle (not lying along the axis) on the body

    (a) is on a circle with center on the axis.

    (b) is on some complex curve.

    (c) is spiral.

    (d) is circular only if the axis passes through the center of mass.

29. A ceiling fan is rotating about a fixed axis. Consider the following statements for the particles not on the axis at a given instant.

    - Statement I: Every particle on the fan has the same linear velocity.
    - Statement II: Every particle on the fan has the same angular velocity.

    The correct statement(s) is (are):

    (a) Statement I only.

(b) Statement II only.

(c) Both statements I and II.

(d) Neither statement I nor II.

Questions 30–32 are concerned with figure A.7. Figure shows a potter's wheel rotating uniformly about a fixed axis. P and Q are two particles on the wheel. P is closer to the axis than Q.

30. The particle P compared to particle Q has a

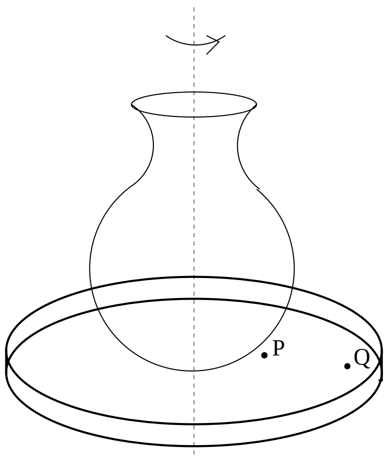

Figure A.7: A potter's wheel (see questions 30-32).

(a) greater angular speed.

(b) smaller angular speed.

(c) greater linear speed.

(d) smaller linear speed.

Now the potter decides to slow down the wheel.

31. Which of the following statements is correct regarding the angular velocity $\vec{\omega}$ of the potter's wheel?

(a) The direction of $\vec{\omega}$ does not change with time.

(b) The magnitude of $\vec{\omega}$ does not change with time.

(c) Both the magnitude and direction of $\vec{\omega}$ change with time.

(d) Neither the magnitude nor the direction of $\vec{\omega}$ changes with time.

32. The direction of the angular acceleration of the potter's wheel at any instant will be

    (a) along the direction of the angular velocity.

    (b) opposite to the direction of the angular velocity.

    (c) perpendicular to the axis of rotation.

    (d) perpendicular to the direction of the angular velocity.

33. A girl is sitting on a stool with her arms outstretched and is rotating with constant angular velocity. The axis of rotation is fixed throughout the motion as shown in figure A.8 (by dotted line). When she folds her arms toward her body she acquires an angular acceleration. Which of the following statements is true regarding the rotational motion of the girl-stool system?

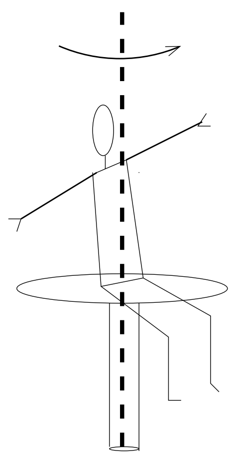

Figure A.8: A girl sitting on a stool, rotating about a fixed axis (see question 33).

(a) There exists an angular acceleration because both the magnitude and direction of the angular velocity of the system changes.

(b) There exists an angular acceleration because only the direction of the angular velocity of the system changes.

(c) A torque ($\vec{\tau}$) acts on the system which results in the angular acceleration $\vec{\alpha}$ as per $\vec{\tau} = I\vec{\alpha}$ (where I is the moment of inertia).

(d) Torque acting on the system is zero.

34. Consider the following statements. Which among them is/are correct?

- Statement I: Angular velocity of a particle depends on the choice of a specified origin.
- Statement II: Angular velocity of a rigid body (rotating about a fixed axis) depends on the choice of a specified origin.

(a) Statement I only.

(b) Statement II only.

(c) Both statements I and II.

(d) Neither statement I nor II.

Questions 35-37 are concerned with figure A.9. Figure shows a wheel spinning about its axis. P is a point on the rim and O is the center of the wheel. We consider accelerations about O.

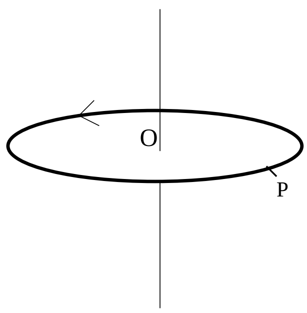

Figure A.9: Schematic of a spinning wheel (see questions 35-37).

35. The wheel is spinning about its axis with constant angular acceleration. The tangential acceleration of the point P

    (a) remains constant in magnitude.

    (b) keeps increasing with time.

    (c) is zero.

    (d) is independent of angular acceleration.

36. The wheel is spinning about its axis with constant angular acceleration (angular acceleration is in the same direction as the angular velocity). The magnitude of the centripetal acceleration of the point P

    (a) remains constant and non zero because $a = \alpha r$ (α and r are constant).

    (b) increases with time.

    (c) is zero.

    (d) is independent of the angular acceleration.

37. The wheel is spinning about its axis with zero angular acceleration. For the point P on the rim there exists

    (a) tangential acceleration only.

    (b) centripetal acceleration only.

    (c) both tangential and centripetal acceleration.

    (d) neither tangential nor centripetal acceleration.

Questions 38–39 are concerned with figure A.10. Figure shows a giant wheel rotating about a fixed axis with **increasing speed.** The axis passes through the center O. G denotes a girl sitting on the wheel.

The arrows numbered (1-4) represent directions to be referred while answering question 38.

38. Which of the four arrows in the diagram best represents the direction of the linear acceleration $\vec{a}$ of the girl at G (relative to the ground)?

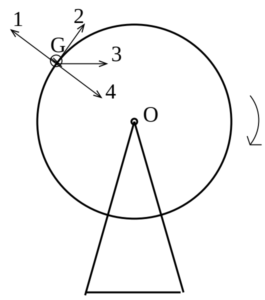

Figure A.10: A giant wheel (see questions 38-39).

(a) 1

(b) 2

(c) 3

(d) 4

39. Let $\vec{r}$, $\vec{a}$ and $\vec{\alpha}$ denote the position vector, linear acceleration and angular acceleration (about O) respectively of the girl at any instant. Which of the following statements is correct?

(a) $\vec{a} = \vec{\alpha} \times \vec{r}$ because the motion is circular.

(b) $\vec{a} = \vec{\alpha} \times \vec{r}$ because angular acceleration is perpendicular to the linear acceleration.

(c) $\vec{a} \neq \vec{\alpha} \times \vec{r}$ because $\vec{a}$ does not denote the acceleration of the center of mass of the whole system.

(d) $\vec{a} \neq \vec{\alpha} \times \vec{r}$ because $\vec{a}$ has a radial component.

Appendix B

Open ended and free response questions: Initial phase of development

B.1 Examples

Below we provide examples of questions given to students during the initial phase of development. The theoretical analyses described in chapter 2 and our experiences while teaching the module on rotational motion helped develop these questions.

1. The second hand of a clock moves from 12'O clock position to 4'O clock position. How would you describe the angular speed of the second hand? What are the different types of units that you will adopt ?

2. How are angular speed and linear speed different? Will linear speed alone suffice to describe motion?

3. What do you understand by the terms 'clockwise and anti-clockwise', often used in the context of angular motion?

4. Suppose a wall clock is transparent. You are free to move around and watch the clock from all sides. Will the motion of the second hand be clockwise always? Explain.

5. A particle is in uniform circular motion. How is the direction of its angular velocity determined ?

6. Consider an oscillating simple pendulum. Is it at rest at any point or time during its oscillation. If yes, what do you think about angular velocity and angular acceleration of the pendulum bob at that point or time? If your answer is no, give reasons for your answer.

7. In the previous question what do you think about linear velocity and linear acceleration.

8. How does the magnitude of angular velocity of an oscillating pendulum bob vary as it moves from one extreme to the other?

9. How does the direction of angular velocity of the pendulum bob vary as it moves from one extreme to the other? Does the direction remain same through out? Explain your answer.

10. The previous question concerns the direction of angular velocity. What do you think about the direction of linear velocity? Will its behavior be identical to angular velocity.

11. How does the magnitude of angular acceleration of an oscillating pendulum bob vary as it moves from one extreme to the other? Does it decrease or increase monotonously?

12. How does the direction of angular acceleration of the pendulum bob vary as it moves from one extreme to the other? Does the direction remain same through out? Explain your answer.

13. Equation $\vec{v} = \vec{\omega} \times \vec{r}$ is always valid. True or False? Explain your answer. If your answer is false, give example of a motion where the equation is not valid.

14. Consider any motion where the object has both linear and angular acceleration. Is the equation $\vec{a} = \vec{\alpha} \times \vec{r}$ valid for the motion. Discuss by explaining each term in the equation.

15. A particle has an angular velocity. Will it necessarily move in a circle? Discuss.

16. A particle is moving in a straight line? Can it have an angular velocity ? Explain.

17. A planet is moving in an elliptical orbit with sun at one of its foci. What do you understand by the direction of linear velocity and angular velocity at a point on the trajectory? Are they same.

18. What do you mean by angular velocity of a rigid body? Illustrate with an example.

19. Consider a point (not lying on axis) on a rotating potter's wheel. Indicate the direction of its a) linear velocity b) angular velocity c) angular velocity d) angular acceleration.

20. Is angular acceleration of a rigid body always directed along its angular velocity? Give an example to illustrate your answer.

21. A ring is rotating with uniform angular acceleration. Discuss whether it has a centripetal acceleration.

Appendix C

Winnowing process of items

C.1 Iterative evolution of items - Examples

Example 1:

Question 1 below was designed initially to probe student understanding of the direction of linear and angular velocity. It was motivated by our interactions with students which revealed that students harbor an array of misconceptions related to the direction of angular velocity and their relationship with linear velocity. We found that students ascribe aspects of linear velocity to angular velocity. Some of them tried to answer the question by resorting to the equation $\vec{v} = \vec{\omega} \times \vec{r}$ and were unaware that the equation was valid only for circular motion. All these observations indicated that the question needs to be refined to be more specific. This iteratively led to the evolution of questions 5 and 6 in appendix A. Question 5 contains distractors like 'angular velocity is tangential to the orbit' which are specific and directly incorporates students' notions, in contrast to the earlier version given below. Question 6 exclusively focuses on student understanding of the validity condition of $\vec{v} = \vec{\omega} \times \vec{r}$. As mentioned earlier our interactions with students have revealed this to be a widespread pitfall.

1. A planet is moving in an elliptical orbit with sun at one of its foci. At a point on its trajectory $\vec{v}$ is the linear velocity and $\vec{\omega}$ is the angular velocity of the planet.

    (a) $\vec{v}$ is parallel to $\vec{\omega}$.

(b) $\vec{v}$ is perpendicular to $\vec{\omega}$.

(c) $\vec{v}$ is at an acute angle with $\vec{\omega}$.

(d) $\vec{v}$ is at an obtuse angle with $\vec{\omega}$.

Example 2:

At an early stage of development of the inventory, only 4 questions were devised in the context of a simple pendulum. Two of them pertained to the angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$ at an extreme position. The other 2 probed the variation of $\vec{\omega}$ and $\vec{\alpha}$ as the pendulum bob moved from one extreme to the other. Question 2 given below was one among them. Our interactions with students unveiled many interesting notions pertaining to student understanding of $\vec{\omega}$ and $\vec{\alpha}$ of the pendulum bob. We have described this pattern of reasoning in chapter 3 using the phrase 'as $\vec{\omega}$ behaves so does $\vec{\alpha}$'. The question 2 below and a similar question on acceleration led to the evolution of items 9-11 and 16-18 listed in appendix A. Questions were crafted to probe magnitude and direction separately unlike the initial version given below. The distractors got modified accordingly to incorporate student ideas uncovered during the interviews with students. Evolution along this thread continued even after phase 1 of administration. As mentioned in chapter 3, a new item (8 in appendix A) was added.

2. Consider an oscillating simple pendulum. As the pendulum oscillates from left extreme to the right extreme, which of the following is true regarding its angular velocity.

   (a) Only the magnitude of angular velocity changes.

   (b) Only the direction of angular velocity changes.

   (c) Both the magnitude and direction of angular velocity change.

   (d) Neither the magnitude nor the direction of angular velocity change.

Example 3:

The initial version of item 19 listed in appendix A was as given below. During interviews with students we found that many chose the correct answer *a*. However their reasoning using the relation $v = \omega r$ is invalid in the context under consideration. Hence we modified the distractor *a* to incorporate this pitfall. As may be seen from appendix A, distractor now reads as '$\omega_A > \omega_B$ because $v = \omega r$, v is constant and $r_A < r_B$.' Also, we observed that many students maintained that $\omega_A = \omega_B$ because linear acceleration is absent. They assumed that the absence of linear acceleration will imply the absence of angular acceleration. Distractor *c* below was modified to incorporate this student idea. It may be recalled that the context was a particle in rectilinear motion with constant velocity (origin not on the line of motion) as shown in figure A.5.

3. The magnitude of angular velocities of the particle at points A and B are ω_A and ω_B respectively. Which of the following relations is true?

   (a) $\omega_A > \omega_B$

   (b) $\omega_A < \omega_B$

   (c) $\omega_A = \omega_B$

   (d) $\omega_A = \omega_B = 0$

Example 4:

Question 4 below was the preliminary version of item 25 in appendix A. The item probed student understanding of the radial and tangential component of velocity of a particle moving in a straight line (with constant velocity) as shown in figure A.5. In our pilot studies we observed many students opting *d*. In the subsequent interviews they made clear their reasoning. Most of them said that since velocity of the particle is constant, its components also will remain constant. We modified distractors so as to incorporate this notion. There also existed another notion that the component of velocity remains constant because no force is acting on the particle. As can be seen from appendix A this aspect was also incorporated.

4. Which of the following statements is correct regarding the linear velocity of the particle?

   (a) Radial component increases.

   (b) Tangential component increases.

   (c) Both radial and tangential components increase.

   (d) Neither radial nor tangential component increases.

Example 5:

Item 26 (see appendix A) was developed to probe student understanding of the angular velocity of a rigid body. An initial version of item 26 is given below as question 5. In our interviews with the students, we observed that many students regard center of mass to be important while describing the angular velocity . Most of them said that center of mass is a representative point of the rigid body. Also they pointed out that an angle is always traced by position vector of a particle. We reframed the distractors to include these student notions pertaining to the center of mass (CoM). Related modifications were made in the distractors to items 27 and 28. As one can see from appendix A these items have distractors involving notions related to CoM.

5. Angle $\Delta\theta$ in the definition of angular velocity of a rigid body $\omega = \Delta\theta/\Delta t$ (rotating about a fixed axis) is

   (a) traced by position vector of any particle on the rigid body with respect to a specified origin.

   (b) traced by a line perpendicular to the axis from any particle on the body.

   (c) determined by angles between two co-ordinate systems, one fixed on the body and the other fixed on ground.

   (d) determined by taking the vector sum of angular velocities of all the particles on the rigid body.

C.2 Minor modifications - Examples

Minor modifications in wording of certain questions or the distractors were made, some after the first phase of administration. They enhanced the clarity of the questions and facilitated easier interpretation during analysis of the response pattern of students.

Example 1:

During the first phase of administration the form of item 9 in appendix A was as given below. The distractors were modified to exactly match those of item 17 which probed angular acceleration. This in turn helped us to compare student thinking about angular velocity and angular acceleration.

1. Throughout the motion of the bob from A to D the magnitude of the angular velocity of the bob

   (a) remains same.

   (b) keeps on increasing.

   (c) keeps on decreasing.

   (d) first increases and then decreases.

Example 2:

Question 2 below is an earlier version of item 11 (see appendix A). The part ‘moves up and down’ in the statement of the question was replaced by ‘going from A to D and from D to A’. This modification was suggested by experts during validation and some of the students during interviews, to remove any ambiguity.

2. The angular velocities of the bob at a point C on the trajectory as it moves up and down are

   (a) equal in magnitude, but differ in direction.

   (b) the same in direction, but differ in magnitude.

(c) different in both magnitude and direction.

(d) equal in both magnitude and direction.

Example 3:

Question 3 below is an earlier version of item 29 in appendix A. During pilot studies some of the students pointed out that the points on the axis of the ceiling fan were at rest. It was not explicitly stated that we exclude these points. In addition, the phrase ‘at any instant of time’ was repeated in all the choices. The phrasing and structure of the question and the ensuing choices were appropriately changed.

3. For a ceiling fan rotating about a fixed axis which of the following statements is true?

   (a) At any instant of time every particle on the fan has the same linear velocity only.

   (b) At any instant of time every particle on the fan has the same angular velocity only.

   (c) At any instant of time every particle on the fan has the same linear velocity as well as the same angular velocity.

   (d) At any instant of time every particle on the fan has different linear velocities as well as different angular velocities.

Example 4:

Question 4 below is an earlier form of item 32 (see appendix A). In the pilot studies distractor *d* did not attract many responses. Also experts and a few students pointed out that ‘direction of motion of wheel’ made little sense. As such we replaced it by the option ’perpendicular to the direction of angular velocity’ in subsequent administrations.

4. The direction of angular acceleration of the potter’s wheel at any instant will be

(a) along the direction of angular velocity.

(b) opposite to the direction of angular velocity.

(c) perpendicular to the axis of rotation.

(d) along the direction of motion of the wheel.

Example 5:

Items 35 and 36 in appendix A served to elicit a case of indiscriminate use of equation $a = \alpha r$ as discussed in chapter 6. The earlier version of item 36 was as given by question 5 below. Distractor *a* was expanded as ‘ remains constant and non zero because $a = \alpha r$, α and r are constant’. This modification was to enable an unambiguous interpretation of student response. Also an expert pointed out that a constant angular acceleration can imply a decreasing centripetal acceleration as well. Though the choices provided has only one answer (*b*) we added a caveat in the latest version of the statement of the item that ‘angular acceleration is in the same direction as angular velocity’, to address the concern.

5. The wheel is spinning about its axis with constant angular acceleration. The magnitude of the centripetal acceleration of the point P

   (a) remains constant and non zero.

   (b) increases with time.

   (c) is zero.

   (d) is independent of angular acceleration.

C.3 Dropped questions - Examples

Example 1:

The experts agreed that checking for a knowledge of units is important, but voted against testing them in a conceptual inventory. As such item 1 below was excluded from the inventory.

1. When the minute hand moves from the 12 O'clock position to the 6 O'clock position, the angular speed of C (see figure A.1) in SI units is

   (a) $\pi/1800$ rad/s

   (b) 1/3600 Hz

   (c) $\pi/60$ rad/s

   (d) $\pi/1800$ Hz

Example 2: Item 2 below was devised to test student understanding of the equations $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a} = \vec{\alpha} \times \vec{r}$. These equations are often used for solving problems in rolling motion, where the particle analyzed is the center of mass of the rigid body. When applied to a general particle $\vec{v}$ and $\vec{a}$ in the equations refer respectively to the tangential component of the velocity and acceleration. We wanted to investigate whether students can understand these intricacies. But in the present case the equations become trivially valid for center of mass since $\vec{v}$=$\vec{a}$=$\vec{r}$=0. Moreover as pointed out by experts, the distractors lack clarity and are clumsy. As such we dropped it and incorporated some of the aspects in other items.

2. Let $\vec{v}$, $\vec{a}$ be the linear velocity and the acceleration respectively of any arbitrary point on a pulley (rotating about a fixed axis with increasing speed). Let $\vec{\omega}$ denote the angular velocity and $\vec{\alpha}$ the angular acceleration of the pulley. Which of the following statements is true?

   (a) $\vec{v} = \vec{\omega} \times \vec{r}$ because $\vec{v}$ is not necessarily the center of the mass.

   (b) $\vec{a} = \vec{\alpha} \times \vec{r}$ because $\vec{a}$ is not necessarily the acceleration of the center of the mass.

   (c) Both relations $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a} = \vec{\alpha} \times \vec{r}$ hold true.

   (d) The relation $\vec{v} = \vec{\omega} \times \vec{r}$ holds true but $\vec{a} = \vec{\alpha} \times \vec{r}$ does not because there exists a centripetal acceleration

Example 3:

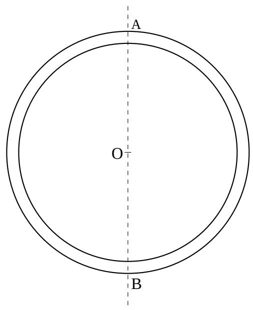

Figure C.1: A circular groove

Question 3 below involved mostly algebraic manipulations. We were careful to avoid items involving calculations. So the question was removed.

3. Figure C.1 shows a circular groove with center O on a horizontal plane. A ball starts at rest from A and moves clockwise with constant angular acceleration $\vec{\alpha}$ in such a way that unobstructed it reaches B with angular velocity $\vec{\omega}$. At the same instant another identical ball is set to motion anticlockwise from B with initial angular velocity $\vec{\omega}$ and constant angular deceleration of magnitude α. The angular displacement of the first ball (about O) when the two balls meet is

   (a) $\pi/4$ rad

   (b) $\pi/2$ rad

   (c) $2\pi/3$ rad

   (d) $3\pi/4$ rad

Example 4:

The reasons given by experts to exclude item 4 below was that it appeared out of syllabus for higher secondary school students.

4. Which of the following statements is **not** true for a rigid body?

   (a) Distance between the different particles constituting the rigid body does not change.

(b) Sum of internal forces acting on particles of the rigid body is zero.

(c) In reality no body is truly rigid.

(d) The number of coordinates required to specify the locations of all particles of a rigid body is 3N, where N is the number of particles.

Example 5:

Though nothing explicitly wrong with question 5 below, experts suggested that the item was too trivial since the body is rotating about a fixed axis. Some of them also pointed out that the distractor *d* was irrelevant.

5. The direction of angular velocity of a rigid body rotating about a fixed axis

   (a) is sometimes along the axis of rotation.

   (b) is always along the axis of rotation.

   (c) is never along the axis of rotation.

   (d) depends on the actual configuration of the rigid body and the directions of forces acting on it.

Example 6:

Question 6 below was developed as part of items on rigid body rotating about a fixed axis. Experts pointed out that all other items strictly adhered to the condition that the axis is fixed. They suggested that the question would be more appropriate in an inventory on rolling without slipping and as such the item was excluded.

6. A solid cylinder is held horizontally by two wires as shown in figure C.2. The wires are wrapped around either ends of the cylinder. The wire ends are tied to hooks on the ceiling. The cylinder falls down as the wires unwind. The direction of angular acceleration of the cylinder

   (a) will be along the direction of gravity.

   (b) will be along the direction of angular velocity of the cylinder.

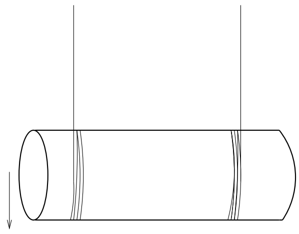

Figure C.2: Solid cylinder rolling down

(c) will be opposite to the direction of angular velocity of the cylinder.

(d) keeps changing with time.

Example 7:

Item 7 was dropped because the same conceptual aspects are being probed by other questions (item 30 in appendix A and to some extend item 29).

7. A girl is sitting on the outermost seat of a merry go round rotating uniformly about a fixed axis. After sometime she shifts to a seat closer to the axis. Her angular speed with respect to a person on the ground

   (a) remains same.

   (b) decreases because the circle which she travels gets smaller .

   (c) increases because her linear speed changes.

   (d) is zero because she is sitting idly on the seat (ignore the time while she was shifting).

Example 8: Question 8 below was dropped because experts pointed out that the item should be part of a separate inventory on non inertial frame of reference at the undergraduate level. It is inappropriate at the higher secondary level as the topic is hardly covered except for passing mention.

8. Let the angular speed of the girl (see question 7 above) with respect to a person on the ground be ω at some instant of time. A boy is sitting on the merry go round closer to the axis than the girl. The angular speed of the girl with respect to the boy will be

(a) ω.

(b) greater than ω.

(c) less than ω.

(d) zero.

Example 9:

The reason for dropping item 9 below was because it concerns rolling without slipping. Experts suggested that the topic needs to be dealt separately as it is complex on its own and has a host of issues that need to be investigated.

9. A wheel of radius R is rolling without slipping on a horizontal surface. The condition for rolling without slipping is $v = \omega R$. Regarding v and ω which of the following statements is correct?

   (a) v is the linear speed and ω the angular speed of any of the particle on the wheel.

   (b) v is the linear speed and ω the angular speed of the center of mass of the wheel.

   (c) v is the linear speed of the center of mass and ω the angular speed of the wheel about the axis passing through the center of mass.

   (d) v is the linear speed of any of the particle on the wheel and ω the angular speed of the wheel about the axis passing through the center of mass.

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